

The Implication Label in the Proof of MPDO Theorem 4.9

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Verdict: local-correction (resolved).

1 The cyclic mismatch

The statement of Theorem 4.9 in [CPGSV16, lines 851–893] numbers SAL together with literal ZCL as condition (ii), Gibbs form together with literal ZCL as condition (iii), the BNT physical-sector structure as condition (iv), and the renormalization fixed-point property as condition (v). The propositions in the Appendix have the following literal component conclusions:

prop2to3 : $(ii) \implies (iv)$,
prop3to4 : $(iv) \implies$ the GSNNCH-form clause of (iii),
prop4to2 : the GSNNCH-form clause of (iii) $\implies$ the SAL clause of (ii),
prop2to5 : $(ii) \implies$ a channel pair between the two- and three-site tensors.

These are the displayed statements at source lines 1740–1817. In particular, **prop3to4** does not conclude the ZCL conjunct of condition (iii), and **prop4to2** does not conclude that conjunct of condition (ii). More seriously, the proof of **prop3to4** invokes **lemmus** from lines 1646–1658 to make the repeated-copy weights independent of the copy label, but **lemmus** assumes ZCL whereas the displayed statement of **prop3to4** assumes only condition (iv). Thus the printed component signature is not itself validated by the following proof; this is a load-bearing instance of proof-path drift. Likewise, **prop2to5** does not itself conclude condition (v). The concluding proof at lines 1819–1825 nevertheless assigns the propositions successively to $(ii) \Rightarrow (iii)$, $(iii) \Rightarrow (iv)$, $(iv) \Rightarrow (ii)$, and $(iii) \Rightarrow (v)$. Thus the discrepancy concerns the proof path and the component conclusions, and is not repaired by changing only the last occurrence from “(iii)” to “(iv)”.

Proposition **3to5** at lines 1510–1563 constructs a pair of channels from one supplied physical-sector factorization. Proposition **prop2to5** then says that the same construction is first performed in each locally orthogonal BNT subspace. Its assertion that the corresponding maps can be combined is the additional

all-BNT step. Lines 1821–1825 then apply the two-to-three-site channels twice to obtain the one-to-two-site maps for the blocked tensor in condition (v).

2 The literal implication (iv) $\Rightarrow$ (v) is false

Condition (iv) constrains each normal BNT representative but places no restriction on the raw weights of its repeated copies. This omission is load-bearing. Let the sole scalar representative be $A = 1$, with two copies of weights 1 and $1/2$. The resulting one-letter tensor has local virtual matrix

$$K^{00} = \text{diag}(1, 1/2).$$

It is exactly a BNT canonical presentation with a global unit-weight copy, generates positive semidefinite periodic operators at every positive length, and its sole BNT representative has nonnilpotent physical-trace transfer. The singleton BNT family has simultaneous one-letter span. The representative itself generates MPDOs, satisfies the distinct-layer equation vacuously, and has the scalar physical-sector factorization with positive neighboring operator and normalized rank-one trace coefficients. Thus all standing hypotheses and every clause of condition (iv) have explicit component witnesses; the two-site block also generates MPDOs.

The two-site block has sole local matrix $(K^{00})^2$. If it satisfied Definition 4.1, trace preservation would make its physical-trace transfer idempotent, hence $(K^{00})^4 = (K^{00})^2$. Taking traces would give

$$1 + 2^{-4} = 1 + 2^{-2},$$

a contradiction. The kernel-checked capstone is `MPOTensor.CaseIIAbsorptionCounterexample.printed_theorem49_iv_to_v_is_false`. Consequently the literal implication (iv) $\Rightarrow$ (v) is refuted. Its classification is *unfaithful statement*; treating projector-controlled outer-BNT assembly as its only missing step is *proof-path drift*. No channel assembly can prove this raw-weight assertion.

This does not refute the source proposition `prop2to5`. That proposition starts from condition (ii), not condition (iv). In the earlier Case-II derivation, lines 1646–1665 use literal zero correlation length to prove that repeated-copy weights are independent of the copy label and then absorb the common weight into the representative. The short proof of `prop2to5` subsequently invokes Proposition `3to5` within each outer BNT sector. Projector-controlled assembly remains a genuine missing statement for this viable (ii) $\Rightarrow$ (v) route.

3 Formal treatment

The formal development therefore does not attach the one-factorization construction to any of the incorrect labels in the concluding paragraph. It cites Proposition `3to5` and the twice-applied channel observation at lines 1821–1825

directly. The resulting theorem is conditional on one supplied factorization of its whole input tensor. It is a *conditional helper*; condition (iv), which supplies data only for each representative, does not meet that hypothesis for a raw repeated-copy tensor. The literal (iv) $\Rightarrow$ (v) claim is therefore not presented as an unfinished direct-sum construction.

Files that use the concluding paragraph carry a *Local fix (cyclic implication labels)* marker and cite this note. This records proof-path drift in the printed proof, including the hidden ZCL use in `prop3to4`. The separate restriction to one supplied factorization is recorded in `docs/paper-gaps/cpgsv17_pf_rank_one.tex`. Issue #6632 is retargeted conceptually to the projector-controlled assembly needed for the source proposition `prop2to5`, after ZCL-derived common-weight absorption; it is not a task to prove the refuted literal implication.

References

- [CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: *Annals of Physics* 378 (2017), 100–149, 2016.