

The Final Traced Subspin in the Definition of $\mathcal{S}_0$

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Verdict: local-correction (resolved).

1 The source sentence

In the proof of Proposition C.7, Appendix C.2 of [CPGSV16], the map $\mathcal{S}_0$ reduces three physical sites to the two outer subspins. The prose at line 1547 of `Papers/1606.00608/MPD0-22-12-17-2.tex` says that it traces the subspins $1r, 2l, 2r, 2l$. The factor $2l$ therefore appears twice, while no subspin of the third site is listed.

2 Correction forced by the diagram

The diagram `Papers/1606.00608/MPD0_TMatrix2.png` shows three consecutive sector factors

$$(L_k \otimes R_k) \otimes (L_l \otimes R_l) \otimes (L_h \otimes R_h)$$

before $\mathcal{S}_0$, and retains only $L_k \otimes R_h$. Hence the traced space is

$$(R_k \otimes L_l) \otimes (R_l \otimes L_h),$$

corresponding to subspins $1r, 2l, 2r, 3l$. The displayed formula for $\mathcal{S}_2$ at lines 1557–1559 confirms the same two retained outer factors. Thus the final occurrence of $2l$ in line 1547 is a typographical error for $3l$.

The formal regrouping uses the corrected equivalence

$$((l_k, r_k), ((l_l, r_l), (l_h, r_h))) \mapsto ((l_k, r_h), ((r_k, l_l), (r_l, l_h))).$$

The corresponding formal declarations are recorded in `TNLean/MPS/MPD0/RFPSubspinMaps`.¹ This correction changes no hypothesis and is local to the ordering of tensor factors.

¹In particular, `MPOTensor.ExplicitEtaOperators.s0RegroupEquiv`, `MPOTensor.ExplicitEtaOperators.s0Map`, and `MPOTensor.ExplicitEtaOperators.s0SectorMap`.

3 Verdict

This is a local correction of a repeated subspin label in the source prose. It does not alter the statement or hypotheses of Proposition C.7.

References

- [CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: *Annals of Physics* 378 (2017), 100–149, 2016.