Physical-Isometry Transport for the Blocked MPDO Fixed Point

2026-07-12

Verdict: scope-restriction (resolved).

1 Source construction and present scope

Proposition `3to5` constructs the channels $\mathcal{T}$ and $\mathcal{S}$ from one supplied physical-sector factorization; see [CPGSV16, Appendix C.2, lines 1510–1563 and 1821–1825]. The observation at lines 1821–1825 composes each channel with an overlapping copy to obtain $\mathcal{T}^2$ and $\mathcal{S}^2$ for the two-site blocking. These formulas are local to one factorization of the whole input tensor. Condition (iv) of Theorem 4.9 instead supplies a factorization for each BNT representative and does not constrain its raw repeated-copy weights. The scalar presentation with weights 1 and $1/2$ satisfies condition (iv) but its two-site block fails Definition 4.1. Thus the one-factor transport theorem is a conditional helper and cannot be extended to the refuted literal (iv)$\Rightarrow$(v) implication merely by combining outer physical supports.

The outer-support construction remains relevant to the viable source proposition `prop2to5`, which starts from condition (ii). In the earlier Case-II derivation, literal ZCL makes repeated-copy weights independent of the copy label and their common value is absorbed. Only after that normalization must the Proposition `3to5` channel pairs be combined along the mutually orthogonal outer physical supports.

The sector-coordinate theorem `NeighboringTraceFactorization.blockTwo_sectorCoordinateTensor_isRFPViaTS` proves the fixed-point identities for

$$\mathrm{blockTwo}(\widehat{\mathcal{K}}),$$

where $\widehat{\mathcal{K}}$ is the tensor in the sector coordinates selected by the physical isometry. Its conclusion is exact in these coordinates, but it does not yet transport the two channels back to $\mathrm{blockTwo}(\mathcal{K})$. It is therefore a scope-restricted intermediate result for one supplied factorization.

2 Transport theorem

In the formal sector factorization, the physical isometry is a square matrix U satisfying $U^\dagger U = I$. Finite dimensionality then also gives $UU^\dagger = I$. The transport theorem proves the closure relations

$$\widehat{\mathcal{K}}_n(X) = U^{\otimes n}\mathcal{K}_n(X)(U^\dagger)^{\otimes n} \qquad (n = 2, 4),$$

with the tensor indices placed in the right-associated conventions used by the channel construction. Transporting $\widehat{\mathcal{T}}^2$ and $\widehat{\mathcal{S}}^2$ through these unitary congruences then preserves complete positivity and trace. The two sector-coordinate closure identities become the required identities for the original blocked tensor.

3 Discharge

The theorem `NeighboringTraceFactorization.blockTwo_isRFPViaTS` formalizes the two- and four-site unitary closure relations, transports the two channels to the original physical matrix spaces, and proves

$$\mathrm{IsRFPViaTS}\big(\mathrm{blockTwo}(\mathcal{K})\big)$$

under the same neighboring-operator trace-factorization hypothesis. This discharges the physical-coordinate transport gap for one supplied whole-tensor factorization. It does not prove the raw (iv)⇒(v) implication, which is false. For the source (ii)⇒(v) route after ZCL-derived common-weight absorption, it supplies the per-factor channel pair whose projector-controlled all-BNT assembly remains a missing statement. That obligation is documented in `docs/paper-gaps/cpgsv17_pf_rank_one.tex` and tracked in issue #6632.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.