

Fibonacci String-Net: the Projector Identity at Positive Length

2026-09-25

Verdict: local-correction (resolved).

1 What the source prints

Section 3.2 of [BMW⁺17] writes the projector matrix product operator on a ring of L sites as $P_L = \sum_a w_a O_a^L$, where $O_a^L = \sum_{i,j} \text{tr}(B_a^{i_1 j_1} \dots B_a^{i_L j_L}) |i_1 \dots i_L\rangle \langle j_1 \dots j_L|$, and requires

$$P_L^2 = \sum_{a,b} w_a w_b O_a^L O_b^L = \sum_a w_a O_a^L = P_L, \quad (1)$$

which “has to hold for all L ”.¹ No lower bound on L is stated. For the Fibonacci theory, Appendix D.1.1 gives two blocks B_1, B_τ of bond dimensions 2 and 3 with weights $w_1 = 1/(1 + \phi^2)$ and $w_\tau = \phi/(1 + \phi^2)$, where $\phi = (1 + \sqrt{5})/2$.²

2 The empty chain

At $L = 0$ the Hilbert space of the chain is one-dimensional and each O_a^0 is the trace of the empty product, the bond dimension χ_a of the block: $O_1^0 = 2$ and $O_\tau^0 = 3$. Using $\phi^2 = \phi + 1$,

$$P_0 = 2w_1 + 3w_\tau = \frac{2 + 3\phi}{1 + \phi^2} = \frac{2 + 3\phi}{2 + \phi} \approx 1.894, \quad (2)$$

which is neither 0 nor 1, so $P_0^2 \neq P_0$. The source derives from (1), required for all L , the fusion rules $O_a^L O_b^L = \sum_c N_{ab}^c O_c^L$.³ These also fail at $L = 0$: the fusion $\tau \otimes \tau = 1 \oplus \tau$ would give $O_\tau^0 O_\tau^0 = 9$ against $O_1^0 + O_\tau^0 = 5$, the unit law $1 \otimes 1 = 1$ would give $O_1^0 O_1^0 = 4$ against $O_1^0 = 2$, and the unit laws $1 \otimes \tau = \tau \otimes 1 = \tau$ would give $O_1^0 O_\tau^0 = 6$ against $O_\tau^0 = 3$. The identity (1) is a statement about rings with at least one site, and the empty chain carries no physical content.

¹Source traceability: `References/1511.08090/AnyonsPEPS.tex`, lines 143–156.

²Source traceability: `References/1511.08090/AnyonsPEPS.tex`, lines 1257–1269.

³Source traceability: `References/1511.08090/AnyonsPEPS.tex`, lines 157–160.

3 The reading adopted

The empty chain is read as a degenerate case the source does not intend. Idempotence, the fusion rule $O_\tau^L O_\tau^L = O_1^L + O_\tau^L$, and the unit laws $O_1^L O_1^L = O_1^L$, $O_1^L O_\tau^L = O_\tau^L O_1^L = O_\tau^L$ are stated under the added hypothesis $L \geq 1$.⁴ This hypothesis is the only restriction recorded here: at every $L \geq 1$ the source's identity (1), with the source's weights, holds for the blocks used in the formalization. Those blocks carry the bare F-symbols and are not identified with the source's diagram tensor; that separate deviation is recorded in [con26].

References

- [BMW⁺17] Nick Bultinck, Michael Mariën, Dominic J. Williamson, Mehmet B. Şahinoğlu, Jutho Haegeman, and Frank Verstraete. Anyons and matrix product operator algebras. *Annals of Physics*, 378:183–233, 2017.
- [con26] The TNLearn contributors. Fibonacci string-net: Entries of the two operator blocks. Paper-gap note bmwshv17_fibonacci_block_entries_provenance, TNLearn, 2026. https://sirui-lu.com/TNLearn/paper-gaps/bmwshv17_fibonacci_block_entries_provenance.pdf.

⁴Results: `FibonacciCompression.fibProjector_mul_self`, `FibonacciCompression.fibonacci_fusion_rule`, `FibonacciCompression.fibOne_mul_fibOne`, `FibonacciCompression.fibOne_mul_fibTau`, `FibonacciCompression.fibTau_mul_fibOne`.