

The Scanner Estimate from the Conclusions of Its Inputs

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Verdict: scope-restriction (open).

1 The source proposition

Proposition 9.2 of the two-dimensional area-law manuscript of September 24, 2026 [Ope26] (`08-scanner.tex`, lines 349–397) states that, under the scan scales and the entropy input $S_\Omega(X) \leq C_e n^{1+e}$, $S_\Omega(Q_j) \leq C_e(nL^e + n)$, for every sufficiently large replica count k some charge round and some common parameter $p_k \in [\epsilon/2, \epsilon]$ give the defect-energy bound

$$E_{\text{def},k} \leq Cn^\ell D^2 W^2 (\log n)^C (\delta_n^{1/8} + \epsilon + n^{-100}) + Cn^{-1000} + o_k(1),$$

with $\delta_n = C_e \epsilon^{-1}(n^{e-\mu} + a^{1/4}(\log n)^C)$, and with the comparator budgets $B_{\text{sh}} \leq C_e(nL^e + n)$, $B_{\text{exc}} \leq CnD$ and $\mathcal{B} \leq CnD(\log n)^C$.

Its proof (lines 399–533) uses Lemma 9.1 (the scan histories), Proposition 7.4 (transport), Proposition 8.1 (comparators), Lemma 2.1 (continuity) and Lemma 2.3 (tails) for the concrete scan.

2 The restriction

The formal statement takes the conclusions of those five results, for one scan at scale n , as the fields of one data package, and proves the step of the source from these conclusions to Proposition 9.2: the integrated entropy balance over the charge rounds, the averaging selection of a round and of a parameter with $\mathcal{Q}(p_k)/(KnD) \leq \delta_n + \rho_k$, the Hölder accounting of the defect energy at the selected point, and the three comparator budgets.¹ Each field transcribes a conclusion of an input and cites its source passage, except that the charge-defect continuity requirement is weakened to the almost-everywhere measurability sufficient for the two integral arguments. The trace-distance field retains a fixed coefficient, as in the source asymptotic bound. In particular, the transported-measure mass equality is required only for interior interpolation parameters.

¹The packaged statement is `TNLean.PEPS.AreaLaw.Scan.scanner_estimate`.

The constants depend only on the fixed exponents and on the constants of the inputs, as in the source.

Two harmless differences in the bookkeeping are recorded. The continuity input bounds the change of the shell entropies by $C\sqrt{Cn^{-500}}(1 + Cn^2 \log q)$, which is $O(n^{-248})$ but not at most n^{-248} itself; the formal budget lemmas carry the error n^{-247} . The marginal parameter is bounded with logarithmic power 12, coming from the crossing count $(\log n)^4$ and the squared dimension term $(\log n)^8$; the source writes an unspecified power C .

3 The fixed coefficient in the trace-distance estimate

The source states that the trace distance is $O(n^{-500})$ [Ope26] (`08-scanner.tex`, lines 400–403). The trace-distance field retains this fixed coefficient: $t \leq Cn^{-500}$, with the common constant $C \geq 1$ chosen before the scale, domain and scan. Here t is one half of the trace norm of the difference of the original and truncated density matrices; it is not a rescaled distance. The former coefficient-one restriction is resolved by retaining C in this field.

For the same truncated ground vector, the perturbation estimate of [Ope26, Proposition 4.5] gives

$$t \leq \sqrt{2\varepsilon/g}, \quad \varepsilon \leq n^{-1000}, \quad g > 0, \quad (1)$$

and (1) yields $t \leq \sqrt{2/g}n^{-500}$. The physical construction must choose its common fixed constant to satisfy $C \geq \sqrt{2/g}$ as well as the other input bounds. Increasing the scale alone does not remove this coefficient.

Here g is the gap of the positive-constraint Hamiltonian before truncation. If the physical Hamiltonian has gap Δ and the positive constraints are divided by their fixed normalization c_s , then $g = \Delta/c_s$. Both the coefficient above and the truncated-gap lower bound $g/2$ use this same normalized gap, rather than Δ .

For any fixed nonnegative A, C and $\log q \geq 0$, the numerical continuity estimate applies uniformly to every $t \leq An^{-500}$ and gives, for all sufficiently large n ,

$$C\sqrt{t}(1 + Cn^2 \log q) \leq n^{-247}. \quad (2)$$

Taking $A = C$ in (2) gives the arithmetic needed for the retained distance field. The threshold depends on the fixed constants and is chosen before the physical instance. This resolves the numerical coefficient restriction; it does not construct the physical distance, continuity, transport or comparator fields. Construction of all inputs from the same original and truncated ground vectors remains open. The physical trace distance keeps its original meaning.

4 Parameter domain of the transported measures

The transported states of Proposition 7.4 are density matrices for $0 < p < 1$ (06-transport.tex, lines 393–400), so the combined Fourier-coherent measures have mass 1/2 there. At $p = 1$ an old terminal leaf has weight $(1-p)w_h = 0$, and at $p = 0$ a new terminal leaf has weight $pw_h q_{c|h} = 0$. The totalized normalized derivative map contains the factor π_j^{-1} and is zero when $\pi_j = 0$. Its transported state and combined measure are then zero. Requiring mass 1/2 for every real p would therefore exclude the literal transported measures.

The mass hypotheses now have the interior domain $0 < p < 1$. The measures remain defined at every parameter; no replacement probability measure is assigned to an inactive leaf. The charge-defect bound and the old- and new-leaf energy estimates use the mass hypothesis only in the interior. The integral over $[0, 1]$ uses the interior bounds and almost-everywhere measurability of the charge defect, since the two boundary points have Lebesgue measure zero. The logarithmic norm remains continuous on $[0, 1]$, and its matching boundary values, initial and final comparisons, and integrated entropy conclusions are unchanged. This domain correction does not construct the transported measures or remove the remaining scanner-input restriction.

5 Parameter regularity from actual transported expectations

Parameter integrability of the exact old-state Fourier-coherent scalar integral follows from the transported expectation theorem. For a continuous real coherent symbol F , let A_F be its coherent average and $Q_F = (A_F + A_F^\dagger)/2$. Positivity of the actual state gives Hermiticity, hence

$$\operatorname{Re} \operatorname{Tr}(\rho Q_F) = \operatorname{Re} \operatorname{Tr}(\rho A_F) = \mathcal{H}_\rho(F). \quad (3)$$

For a nonzero supplied prevector the accepted parameter-measurability result and bound $|J_h(F)| \leq \|Q_F\|/2$ give integrability on $(0, 1)$ and $[0, 1]$. For a zero prevector the actual filtered vector and trace-adjoint state are zero. This argument includes zero copies and does not assume symmetry. Finite sums with the original fixed good-history predicate and weights give the regularity of the actual charge defect.²

The scanner package now asks only for almost-everywhere measurability of its scalar charge defect. Its existing nonnegativity and upper bound derive interval integrability. The derivative telescope retains closed-interval continuity of the logarithmic norm, differentiation at every interior point, and the pointwise

²The source declarations are `TensorPower.ReplicaTransport.TransportData.intervalIntegrable_oldFourierCoherentIntegral`, `TensorPower.ReplicaTransport.TransportData.intervalIntegrable_sum_oldFourierCoherentIntegral`, and `TNLean.PEPS.AreaLaw.Scan.CollarScan.intervalIntegrable_actualChargeEntropyDefect`.

lower bound; its integral comparison requires only an integrable lower bound. No absolute continuity assumption is added. For exact selection, finite averaging chooses a round whose integral is at most the threshold times interval length. If the original pointwise defect were strictly above that threshold everywhere, the integral of their integrable difference would be strictly positive, a contradiction. Thus selection preserves the exact non-strict inequality, the same round and parameter, all constants, and the existing remainder.

This derives integrability rather than the manuscript's asserted parameter continuity. The finite-sum theorem is connected below to literal fill and charge rounds by their exact scalar identities. The scalar expectation argument uses the accepted dependency. The literal-measure construction and its specializations now use the accepted canonical measure theorem at the updated dependency revision. The full scanner package, common physical prevector and comparators, and lattice ground-state construction remain separate obligations.

6 Finite history construction now available

The physical partitions, independent uniform offsets and charge choices, padded row fills, and fixed-length labelled charge lists are now constructed as a finite classical history process. For positive offset and slot counts, the weights are normalized probabilities. Blank and previously assigned fill slots consume the deterministic schedule; charges preserve the nominal fronts. The actual unassigned sites lie beyond every completed row.

For the induced lattice graph and actual ambient target depth, the ambient row bound, bounded anchor multiplicity, and the source cut clearance give on every good history a unique slot for each splitting radius- r_0 ball. Its conditional probability is exactly $1/(2[C_1 nD])$, bounded below by $[2(C_1 + 1)nD]^{-1}$, and the evaluated extended history moves exactly its old unassigned part. Goodness bounds assigned sites of A ; it does not constrain the exterior sites initially on the far side. The probabilities are classical history weights, not the transported state measures. Nonnegative costs depending on band, receiving side, and moved subset also satisfy the corresponding expected-cost lower bound, by summing the disjoint labelled events within each band and then summing the bands. The identification of these costs with the transported conditional entropies is established below.³

These results do not construct the entire data package. The actual old middle lies in the compact truncation set, so the existing crossing-support result now identifies every splitting variable-radius truncated designated support with its radius- r_0 charge ball. The sampling conclusion therefore applies to the designated support itself, and the expected-cost lower bound also holds for its band-side-label incidences.⁴ The unconditional assigned-site lead is now derived from

³The physical sampling statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.good_split_sampling_domainGraph`.

⁴The composition is `TNLean.PEPS.AreaLaw.Scan.CollarScan.good_designated_split_sampling_domainGraph`, using `TNLean.PEPS.AreaLaw.truncationRadius_eq_of_mem_crossingL`

the actual evaluator. Together with consumed-row coverage, it confines each receiving–middle incidence to an anchor interval of width $D + 3r_0 + 1$. Actual affine offset counting, exact uniform offset marginals, and the separation of band centers give total incidence weight at most $10D/m$, separately for old and new histories and for their common convex mixtures in fill and charge rounds. The designated support is identified with its base-radius ball from the actual middle’s containment in the truncation set. No good-history or state-dependent independence assumption enters this dilution estimate.⁵ The bounded incidence event contains the terminal metric split event of `06-transport.tex`, lines 336–352. It is not an arbitrary entropy cut such as $U = P_0 \setminus X$, and it is not yet an identification of the arbitrary split predicates stored in the scanner data package with concrete transported-state measures.

The finite ancestry probability argument is now derived from the same actual history process. Every bad endpoint has a strictly time-ordered selected-charge path. The physical cut clearance keeps its predecessors in A , including for the far side. If its length is s and the tested pair time is k , then $D < 4r_0s$ and every path time lies in the last $4n(r_0s + 1) + 1$ pair indices. Independent selected-coordinate probabilities, exact binomial time counting, and the lattice diamond count give the finite tail. Union over actual prefix times, both sides, all bands, and compact endpoints, and averaging the initial offsets, give

$$2K(|T| + nL)(N + 1) \sum_{\substack{0 \leq s \leq N \\ D < 4r_0s}} \rho^s, \quad \rho = \frac{9e nr_0 \mu(1 + 2(2r_0)(2r_0 + 1))}{2[C_1 nD]}. \quad (4)$$

The compact endpoint bound is proved from the actual ambient layers, so no exterior-domain volume enters. Label paths suffice for the spatial count; predecessor witnesses need not be counted because they do not enter a selected coordinate event. The ratio in (4) is sharper than the source’s loose predecessor-inclusive estimate.⁶

The rounded source scales now give arbitrary inverse-power decay of this actual all-prefix probability, uniformly in the finite domain, target, label family, and scanner. The threshold is chosen from the fixed exponents and constants alone. In particular, the actual bad-history probability is at most n^{-200} for sufficiently large n .⁷ This closes the classical asymptotic probability estimate. The remaining geometric conclusions and transported-state estimates remain separate; identifying quantum entropy costs and constructing the entire scanner data package are not consequences of a probability bound alone.

The physical nesting of distinct bands is now derived for independent actual histories and times. Actual designated receiving–middle incidences satisfy the

abels.

⁵The charge and fill mixture statements are `TNLean.PEPS.AreaLaw.Scan.CollarScan.designated_split_mixture_dilution_domainGraph` and `TNLean.PEPS.AreaLaw.Scan.CollarScan.fill_split_mixture_dilution_domainGraph`.

⁶The all-time finite statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.sum_historyWeight_bad_through_geometric_domainGraph_le`.

⁷The result is `TNLean.PEPS.AreaLaw.Scan.eventually_bad_history_probability_le`.

full terminal split rule in both completed and pre-charge statuses. Their labelled occurrence count is at most $10KnD\mu$, with anchor multiplicity μ , under the expanded numerical margin $4(D + 2r_0) \leq m$. Actual completed and pre-charge statuses now have deterministic whole-row comparison prefixes from one nested family. Under $D \geq 1$, $r_0 \leq D$, $4D \leq m$, $8Km \leq L$, the ambient row bound, and the respective $k \leq nm$ or $k + 1 \leq nm$ horizon, the sets U and UY differ from their comparison prefixes by at most $3nD$ sites. The comparisons are inclusions in opposite directions: the near comparison is contained in U , while UY is contained in its far comparison. The partially consumed row is counted in the error. With the source cut clearance, physical undirected cut-edge counts are at most $20nD$ for U and UY , $40nD$ for Y , and $16nD$ for P_0 and V . These bounds are derived from the actual history evaluator and physical lattice geometry, without a prefix or cut-count hypothesis.⁸ For positive factor dimensions and a nonnegative metric parameter, actual band nesting now implies commutation of the band metrics on the symmetric replica space. The complementary metric substitution remains restricted to the symmetric projection, and every intervening factor preserves that subspace. The augmented quantum-region moves agree with the actual physical charge transitions. Cross-band commutation holds for independent histories and all completed/pre-charge status pairs, including every pair of terminal leaves of a charge round.⁹ This commutation does not identify the tree inputs with transported quantum states or establish their entropy-cost identities.

The initial-offset, charge-choice, and recursive history mean trees are now constructed, rather than supplied. Their leaf weights equal the actual uniform history law $m^{-K}(2M)^{-Kk}$, with charge weights $(2M)^{-K}$ and deterministic fill weight one. Substitution constructs the next matrix mean tree without reassociation, and the old and new interpolation endpoints agree with the corresponding tree evaluations for positive definite inputs.¹⁰ These tree identities do not construct the initial quantum prevector or identify the history-tree inputs with its transported quantum states. The pointwise entropy sampling theorem specializes the actual nonnegative move-cost inequality to regional conditional mutual information. Every trial set is first intersected with the old middle; the receiving side and the old pure vector are kept fixed while charge choices are averaged. Disabled choices and empty moved sets have equal zero entropy, without identifying their distinct move constructors. Support compatibility derived from the

⁸The physical results are `TNLean.PEPS.AreaLaw.Scan.CollarScan.state_prefix_approximation_domainGraph`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.oldChargeState_prefix_approximation_domainGraph`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.state_cut_bounds_domainGraph`, and `TNLean.PEPS.AreaLaw.Scan.CollarScan.oldChargeState_cut_bounds_domainGraph`.

⁹These conclusions are `TensorPower.ReplicaTransport.bandMetric_commute_on_symmetric_of_nested`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.quantumChargeMove_apply`, and `TNLean.PEPS.AreaLaw.Scan.CollarScan.chargeTransportData_crossBandCommute`.

¹⁰The construction and identities are in `TNLean.PEPS.AreaLaw.Scan.historyMeanTree`, `TNLean.PEPS.AreaLaw.Scan.historyMeanTree_weight`, `TNLean.PEPS.AreaLaw.Scan.historyMeanTree_eval_succ`, `TNLean.PEPS.AreaLaw.Scan.charge_interpRoot_zero`, and `TNLean.PEPS.AreaLaw.Scan.charge_interpRoot_one`; the same module gives the singleton fill trees and their endpoints.

actual geometry identifies the incidence sum with terminal split entropy. The pointwise inequality is then integrated against the same actual old transported state for all conditional choices, retaining the exact Fourier and coherent integrals. The resulting entropy gain dominates the good-history split defect with coefficient $[2(C_1 + 1)nD]^{-1}$. The history weight in this derivative expression has no terminal old-edge factor $1 - p$. For a nonzero symmetric common prevector and an interior interpolation parameter, the combined integral has mass $1/2$.¹¹ A direct inequality for the canonical integrals does not alone construct the scanner's measure-valued fields. The construction below uses the canonical measure result. An initial prevector, typical sectors, comparator budgets and the complete scanner data remain open.

The missing direct near-far case is now excluded from the actual geometry. The near side lies in the truncation set, so a designated support meeting both receiving sides is a radius- r_0 graph ball. Clearance places the ball inside A ; the unconditional lead bounds and separated fronts then give a contradiction. Thus every support, without a supplied middle-incidence premise, is either contained in one physical part or satisfies the terminal split rule. No good-history condition is used.

The auxiliary factors remain outside the physical supports. The augmented finite-tree compatibility condition therefore holds for every old and new leaf of an actual charge round.

The deterministic fill uses the same recursive history tree and a singleton conditional tree. Its move transfers exactly the scheduled singleton's unassigned part, and the new partition is the following charge's old partition. Admissibility, cross-band commutation and designated-support compatibility are derived from the actual scan. The adjacent fill and charge roots agree without reassociation of matrix means.

The physical cardinality and logarithmic-dimension bounds are checked. For physical dimension $q \geq 1$, the explicit cap is $\ell = 1 + (2r_0 + 1)^2 \log q \geq 1$. Fills transfer at most one site and charges at most $(2r_0 + 1)^2$ sites. A designated support has the same bound when its actual partition is nonconstant on that support: initial far-side persistence and $8Km \leq L$ give a physical singleton cut crossing it, hence reduce its truncation radius to r_0 . No support compatibility, good-history, row-bound, clearance, or time-horizon assumption is needed for this reduction. Unsplit supports remain unbounded. For the rounded radius $r_0 = \lceil C_r(\log n)^2 \rceil$, $C_r \geq 0$, the checked eventual estimate is $\ell \leq [1 + (2C_r + 3)^2 \log q](\log n)^4$, with threshold chosen independently of the domain, scanner, labels, and histories.

The quantum transport statements identify moved subsystems with embedded physical middle differences, so auxiliary-factor dimensions do not enter their cap. A designated support has this cap when its actual split-leaf set is nonempty; no bound is imposed on supports that never split.

The actual truncated positive constraints, extended by the identities on both

¹¹The integrated inequality is `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualChargeEntropyDefect_le_entropyGain_domainGraph`.

auxiliary factors, are positive contractions with their designated physical supports. Their replica sum is exactly the copy mean of the augmented truncated Hamiltonian. For both actual fill and charge rounds, Proposition 7.4 now gives the logarithmic-norm derivative, entropy lower bound and energy upper bound for $0 < p < 1$, and the integrated entropy bound for $0 \leq p_0 \leq p_1 \leq 1$.¹² They require a supplied nonzero symmetric prevector and the smallness condition $a\ell \leq c_0$. Only the energy bound additionally requires that prevector to be an eigenvector of the actual augmented truncated replica Hamiltonian.

Collecting all $2nm$ actual rounds into one finite family, with history length $\lfloor j/2 \rfloor$, even fills and odd charges, one band count K , and one augmented energy family, gives the finite-family form of Proposition 7.4 with common nonnegative errors chosen before the round, replica count, supplied vector, interpolation parameter, and eigenvalue. Its entropy error has the all-count bound $0 \leq \beta_k \leq C_\beta \log(k+2)$ with $C_\beta \geq 0$, and its nonnegative energy remainder tends to zero.¹³ The auxiliary dimensions remain arbitrary and fixed before these errors. Write $\bar{H}_k$ for the actual augmented truncated replica Hamiltonian at replica count k and pre for the supplied nonzero symmetric vector. The energy conclusion assumes a real eigenvalue E_0 with $\bar{H}_k \text{pre} = E_0 \text{pre}$. At zero copies, $\bar{H}_0 = 0$, so this premise forces $E_0 = 0$; no identification with a fixed physical ground energy follows.

The logarithmic norm differences now telescope over the computed $2nm$ actual fill and charge rounds. The successive root equalities follow from the actual recursive history trees, and the final root is the completed history root after nm charges. This algebraic identity allows any supplied vector, an empty round family when $n = 0$, and zero replicas. Under the same physical and geometric hypotheses as the common-error theorem, which retain $n > 0$, a supplied nonzero symmetric vector therefore satisfies the full-round integrated entropy bound with exactly $2nm(C_{\text{ent}}kaKa^{1/4}\ell^{e_{\text{ent}}} + \beta_k)$ as its accumulated error. The parameter integrability of the entropy gain is derived from admissibility and actual cross-band commutation.¹⁴ No initial or final norm comparison or energy eigenvector equation is assumed in this entropy bound. This does not select a charge round or parameter, nor construct the physical prevector or comparator budgets.

The initial physical prevector, typical sectors, comparator estimates, the scanner's measure-valued fields and the complete scanner data construction remain open.

¹²The actual specializations are `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualFill_transport_domainGraph` and `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualCharge_transport_domainGraph`.

¹³The finite-family statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualRounds_transport_domainGraph`.

¹⁴The algebraic and integrated statements are `TNLean.PEPS.AreaLaw.Scan.CollarScan.su_m_actualRoundData_logNormSq_sub` and `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualRounds_entropy_integral_le_domainGraph`.

7 Literal measure, scalar identities and parameter regularity

The canonical measure theorem sets the one-copy domain to its actual unit sphere and takes the old and new measures from the very same terminal transport states, supplied replica vector, and root path. No classical history or conditional-choice weight is inserted inside a measure. For $a \geq 0$, the source identifies integrals of continuous ambient symbols restricted to the sphere with the existing Fourier-coherent integrals. Fixed-parameter finiteness needs no analytic certificate or sign assumption on a . For $a \geq 0$ and a supplied nonzero symmetric vector, the interior mass is $1/2$; this includes zero copies with a nonzero scalar vector. Inactive old and new leaves at $p = 1$ and $p = 0$ retain zero measure without any vector or sign assumption. For $a \geq 0$ and a nonzero symmetric vector, the complementary active leaves have mass $1/2$. No endpoint measure is replaced by an arbitrary probability.

For $a \geq 0$, the same source identifies the round's choice gain and charge defect with the existing entropy gain and actual good-history defect for every real p . Under $a \geq 0$, the energy identity retains both split-probability factors and is restricted to $p \in [0, 1]$: outside that interval the transport tree clamps p , whereas the round's displayed weights use $1 - p$ and p directly. The actual augmented replica-energy operator is retained with the same filtered vector.¹⁵

For $a \geq 0$ and each fixed natural replica count, the literal fill and charge defects are almost-everywhere measurable on $(0, 1)$ and interval integrable on $[0, 1]$. Their pointwise equality with the finite old-state sum transfers the preceding regularity theorem without any probability normalization of the leaf measures. The coherent symbol is

$$F_h(\theta) = \sum_i \eta_{i,(h,\text{old})}(\theta), \quad \mathcal{Q}_k(p) = \sum_{h \text{ good}} w_h J_{h,k,p}(F_h). \quad (5)$$

An unsplit term vanishes by its defining entropy sum. Both the generic round and the actual fill and charge specializations retain the same supplied family ψ_k ; they require neither a nonzero vector nor symmetry nor positive replica count. The two original endpoint values are retained by closed-interval integrability. No parameter continuity or absolute continuity statement follows, and no measurability assumption on a measure-valued family is inserted.¹⁶

¹⁵The declarations are `TNLean.PEPS.AreaLaw.Scan.transportScanRound`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualFillScanRound`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualChargeScanRound`, and `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualChargeScanRound_chargeDefect`.

¹⁶The literal-round regularity declarations are `TNLean.PEPS.AreaLaw.Scan.aestronglyMeasurable_transportScanRound_chargeDefect`, `TNLean.PEPS.AreaLaw.Scan.intervalIntegrable_transportScanRound_chargeDefect`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.aestronglyMeasurable_actualFillScanRound_chargeDefect`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.intervalIntegrable_actualFillScanRound_chargeDefect`, `TNLean.PEPS.AreaLaw.Scan.CollarScan.aestronglyMeasurable_actualChargeScanRound_chargeDefect`, and `TNLean.PEPS.AreaLaw.Scan.CollarScan.intervalIntegrable_actualChargeScanRound_chargeDefect`.

This is not a construction of the complete scanner data. The canonical measure theorem is accepted in QICLean at revision `acfb5c685e732f987a5a96a5c079ba7f821c4715`; the declared dependency and matching manifest revision now select that commit. The toolchain, Mathlib and all other dependency pins are unchanged. Fixed-parameter finiteness is not parameter continuity. The endpoint norm and integrated entropy conclusions are unchanged. The physical prevector, Bell or typical sector and comparator remain separate (the prevector/comparator task is issue 8753); the lattice ground-state task is issue 8979. In particular, no zero-copy energy equation with a nonzero supplied vector implies an arbitrary prescribed ground energy.

8 One-round integrated charge defect

Under the actual charge sampling and transport hypotheses, set $\alpha = [2(C_1 + 1)nD]^{-1}$ and $\ell = 1 + (2r_0 + 1)^2 \log q$. Write $\mathcal{Q}(p)$ for the actual charge defect, $\mathcal{G}(p)$ for the entropy gain of Proposition 7.4, and $N^2(p)$ for the squared norm of the transported filtered vector at parameter p ; C_{ent} and e_{ent} are the universal constants of the actual charge transport estimate. The pointwise inequality $\alpha \mathcal{Q}(p) \leq \mathcal{G}(p)$ combines with the integral conclusion of Proposition 7.4. For a supplied nonzero symmetric vector, $a > 0$, $a\ell \leq c_0$, every natural replica count r , and $0 \leq p_0 \leq p_1 \leq 1$, the resulting bound is

$$ra\alpha \int_{p_0}^{p_1} \mathcal{Q}(p) dp \leq \log N^2(p_0) - \log N^2(p_1) + (p_1 - p_0)(C_{\text{ent}}raKa^{1/4}\ell^{e_{\text{ent}}} + \beta_r). \quad (6)$$

Here the universal constants precede the geometric and Hamiltonian choices, and $\beta_r = O(\log(r + 1))$ is chosen before the replica count and vector. The charge defect and entropy gain are interval integrable; continuity of either scalar is not required. The two error terms retain their signs and the interval-length factor. Equal endpoints give equality, and zero replicas require no separate positive-count assumption.¹⁷ This removes an assumed one-round charge integral bound from the construction. The all-round estimate below combines actual root identities with one common error sequence. Identifying the supplied vector in the actual scanner coordinates and proving its initial and final norm comparisons remain separate.

9 All-round integrated charge defect

The charge-sampling and full-round entropy estimates combine before any initial or terminal norm comparison is applied. This is the intermediate step in the proof of Proposition 9.2 of [Ope26]. Write $\mathcal{Q}_{s,k,\text{pre}}$ for the literal good-old-history defect at charge stage s and replica count k , and $N_{s,k,\text{pre}}^{\text{fill}}$ for the norm of the

¹⁷The one-round consequence is `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualCharge_integral_bound_domainGraph`.

supplied vector after applying the inverse quarter power of the completed-history root after s charges. Under the joint geometric sampling and actual transport hypotheses, put $\alpha = [2(C_1 + 1)nD]^{-1}$ and $\ell = 1 + (2r_0 + 1)^2 \log q$. For the same nonzero symmetric supplied vector, every $k \geq 0$ and every $0 \leq p_0 \leq p_1 \leq 1$ satisfy

$$ka\alpha \sum_{s=0}^{nm-1} \int_{p_0}^{p_1} \mathcal{Q}_{s,k,\text{pre}}(p) dp \leq \log(N_{0,k,\text{pre}}^{\text{fill}})^2 - \log(N_{nm,k,\text{pre}}^{\text{fill}})^2 \quad (7)$$

$$+ 2nm(C_{\text{ent}}kaKa^{1/4}\ell^{e_{\text{ent}}} + \beta_k). \quad (8)$$

The universal constants $c_0, C_{\text{ent}}, e_{\text{ent}}$, with $c_0 > 0$, precede the physical data and require $a\ell \leq c_0$. After the physical and auxiliary dimensions are fixed, one sequence satisfying $0 \leq \beta_k \leq C_\beta \log(k+2)$, with $C_\beta \geq 0$, is chosen before every replica count, vector and interval. The full error is retained even when $p_0 = p_1$. Neither $k > 0$ nor $K > 0$ is assumed.¹⁸

The sign used to add fills is conditional mutual information nonnegativity from strong subadditivity. Positive old-state Fourier-coherent integration preserves it. Integrating sampling on the retained interval, enlarging the non-negative gain integrals to the full interval, and inserting each charge stage at its odd round gives (8). Literal scalar integrability suffices; parameter continuity is not assumed.

The result retains actual charge defects and does not assume their integrated bound as part of completed scanner data. The physical Schmidt–Bell prevector must still be identified with the supplied vector in the actual root and metric coordinates. Its initial norm floor, terminal comparator estimate, and the subsequent physical selection and energy estimates remain separate requirements for the complete scanner proposition.

10 Actual charge energy and uniform old-history tails

The finite-leaf energy calculation of [Ope26] (`08-scanner.tex`, lines 416–440 and 492–511) now has a supplied-vector statement for an actual charge round. Let $s+1 \leq nm$, $n \geq 2$, $1 \leq D$, $r_0 \leq D$, $4(D+2r_0) \leq m$, $8Km \leq L$ and $C_1 > 0$. Retain the induced physical graph, ambient target-distance depth, actual row and cut-clearance bounds, and anchor multiplicity at most $\mathbf{m}$. For $a \geq 0$, a nonzero symmetric supplied vector ψ_k and $0 < p < 1$, write $\mathcal{Q}_k(p)$ for the actual good-old charge defect and $\mathcal{E}_k(p)$ for the literal energy-error sum in Proposition 7.4. Set $A = 10KnD\mathbf{m}$, $\ell = 1 + (2r_0 + 1)^2 \log q$, and let $b = \sum_{h \text{ not good}} w_h$ be the actual classical bad-old probability. Then

$$\mathcal{E}_k(p) \leq \frac{10D}{m} \left((1-p)A^{7/8} \mathcal{Q}_k(p)^{1/8} + \frac{A(2\ell)^{1/8}}{2} ((1-p)b + p) \right). \quad (9)$$

¹⁸The actual sum bound is `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualCharges_integral_bound_domainGraph`.

The split-label capacity A is deterministic. It is distinct from the mass $1/2$ of each actual Fourier-coherent leaf measure. Those mass identities are derived for this same supplied vector; norm one is not an assumption. The Hölder argument divides by neither the capacity nor the total good-history weight. It therefore includes $K = 0$, $A = 0$, zero good-history weight and zero replicas. The bound $\eta \leq 2\ell$ uses only the physical support and is independent of both auxiliary dimensions.¹⁹

At the fixed scanner exponents $\ell_{\text{sc}} = 10^{-5}$, $\kappa = 10^{-6}$, $\mu_{\text{sc}} = 1/4$, and the literal rounded scales, a single threshold now derives $4(D + 2r_0) \leq m$ and replaces b by n^{-P} for any fixed real P . Its dependencies are the radius and charge-slot constants, compact-target coefficient, anchor multiplicity bound and P . It precedes the domain, labels, scanner, charge round, physical and auxiliary dimensions, replica count, supplied vector and common interpolation parameter.

The old-prefix probability is exactly its full-history marginal. Failure of old goodness implies failure of goodness through the full history, so the accepted all-time tail applies without another factor counting rounds. The stronger counting margin follows from $r_0 \leq D$ and $12\lceil n^\kappa \rceil \leq \lfloor n^{\mu_{\text{sc}}} \rfloor$, eventually for this fixed positive exponent gap. Consequently (9) holds with b replaced by n^{-P} uniformly over all $s + 1 \leq nm$, at the same supplied vector and $a/2$.²⁰

For fixed physical dimension $q \geq 1$ and arbitrary real transport coefficient C_{en} and exponent e_{en} , the rounded scales give a further uniform coefficient estimate. There are $C \geq 1$, $C_l \geq 0$ and N , chosen before the physical scanner, charge round, auxiliary dimensions, vector, weight and interpolation parameter, such that for $n \geq N$, $W \geq 0$ and $a = W/K$,

$$|C_{\text{en}}|a^2\ell^{e_{\text{en}}}\mathcal{E}_k(p) \leq CW^2n^{\ell_{\text{sc}}}D^2(\log n)^{C_l} \left((1-p) \left(\frac{\mathcal{Q}_k(p)}{KnD} \right)^{1/8} + (1-p)n^{-P} + p \right). \quad (10)$$

Here $K > 0$ is derived from the eventual rounded-scale facts, and $\mathcal{Q}_k(p) \geq 0$ is derived from the literal leaf measures and nonnegative split entropies. The exact dimension coefficients after normalization are $10(10\mathbf{m})^{7/8}|C_{\text{en}}|\ell^{e_{\text{en}}}$ and $50\mathbf{m}2^{1/8}|C_{\text{en}}|\ell^{e_{\text{en}}+1/8}$. A common logarithmic-power bound absorbs them, including negative real exponents. No division by anchor multiplicity is used. The statement retains $W = 0$, $\mathbf{m} = 0$, $q = 1$ and $k = 0$.²¹

This remains a supplied-vector charge estimate with unselected defect $\mathcal{Q}_k(p)$. It does not assert the full energy expectation in lines 507–511. Identifying the physical Schmidt–Bell prevector with the actual root and metric coordinates,

¹⁹The actual supplied-vector statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualCharge_energyError_le_domainGraph`.

²⁰The classical bound and its energy consequence are `TNLean.PEPS.AreaLaw.Scan.exists_badOldHistory_threshold` and `TNLean.PEPS.AreaLaw.Scan.exists_actualCharge_energy_threshold`.

²¹The literal nonnegativity and fixed coefficient estimates are `TNLean.PEPS.AreaLaw.Scan.CollarScan.actualChargeEntropyDefect_nonneg` and `TNLean.PEPS.AreaLaw.Scan.exists_actualCharge_energy_coefficients`.

selecting a small charge defect, and applying comparator estimates remain separate. No complete scanner construction is asserted.

11 Truncated physical marginal estimates

The moment and tail estimates of [Ope26, Lemma 3.1] are established in `TNLean/PEPS/AreaLaw/Scan/ActualMarginalTails.lean` for every actual regional state of one truncated ground vector. The terms are the actual positive constraints centered at the original unit vector Ω , truncated near the scanner's compact set. Writing $\tilde{H} = \sum_i \tilde{h}_i$, the physical ground-state conditions are

$$\|\tilde{\Omega}\| = 1, \quad (11)$$

$$\tilde{H}\tilde{\Omega} = e\tilde{\Omega}, \quad (12)$$

$$\tilde{H} - eI \geq (g/2)(I - |\tilde{\Omega}\rangle\langle\tilde{\Omega}|), \quad (13)$$

where $g = \Delta / \max\{1, J(\|f_{1,\Delta/2}\|_1 + 1)\}$. Here Δ is the spectral gap of the original Hamiltonian, J bounds the operator norms of its terms, and $f_{1,\Delta/2}$ is the time kernel of the quasi-local spectral filter of [Ope26, Lemma 4.2] with exponent 1 and frequency cutoff $\Delta/2$; $\|f_{1,\Delta/2}\|_1 = \int_{\mathbb{R}} |f_{1,\Delta/2}(t)| dt$ is its L^1 norm, and $\max\{1, J(\|f_{1,\Delta/2}\|_1 + 1)\}$ is the normalization of the positive constraints in [Ope26, Section 4]. The resulting tail parameter is $2/g$, and the entropy center is the entropy of the actual marginal of this same $\tilde{\Omega}$ for every cut. The exact budget preserves every original label, including repeated supports and zero terms; complementary cuts have the same budget, and the empty and full cuts have budget one.

The pair $(e, \tilde{\Omega})$ in (13) is now constructed from Proposition 4.5. Fix R , $J \geq 0$, $\Delta > 0$ and $C_0 \geq 0$. One constant $C_{\text{tr}} > 0$ is chosen before the local dimension, domain, interaction, original ground vector and scan. For every such instance, the hypotheses retained are the original Hamiltonian's gapped ground-state condition, the induced domain graph, anchors in their original admissible supports, $n \geq 2$, the radius equality $r_0 = \lceil C_{\text{tr}}(\log n)^2 \rceil$, and $|S_0| \leq C_0 n^2$. The chosen radius constant is independent of the scan's charge-slot constant C_1 .²² The graph, set, radius and centered constraints agree term by term with the physical truncation theorem. Its unit-norm, eigenvector and positive semidefinite gap conclusions therefore give the actual physical $\tilde{H}$ in (13). The resulting single pair is chosen before all cuts. With $\epsilon_n = \min\{n^{-1000}, g/4\}$, retain the quantitative conclusions of Proposition 4.5 for this same pair:

$$0 \leq e \leq \epsilon_n, \quad (14)$$

$$\|\tilde{\Omega} - e^{i\theta}\Omega\| \leq 2\sqrt{\epsilon_n/g}, \quad (15)$$

$$\frac{1}{2} \| |\tilde{\Omega}\rangle\langle\tilde{\Omega}| - |\Omega\rangle\langle\Omega| \|_1 \leq \sqrt{2\epsilon_n/g}. \quad (16)$$

²²The existential statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.exists_truncated_reducedState_moment_tail_bounds`.

Here θ is a suitable real phase. This same vector satisfies both marginal inequalities for every cut; no truncated spectral, energy-smallness, closeness or tail hypothesis is supplied. In particular, $e \leq n^{-1000}$ and $e \leq g/4$ hold without a second choice of ground vector.

No kernel equation for each individual term is asserted, and e is not assumed to vanish. The original Ω remains the filter center. The projector gap concerns the physical sum before either auxiliary identity extension; a nontrivial spectator can destroy the one-vector projector gap. Empty domains, local dimension one, empty truncation sets and retained disconnected components are allowed. An arbitrary cut inside a retained component can still have crossing labels.

The geometric status estimates now give one common number $1 \leq \mathcal{B} \leq C_B n D (\log n)^{12}$ for the actual five regions XU, V, Y, U, UY , at every completed status with $k \leq nm$ and every pre-charge status with $k+1 \leq nm$. Initial far assignments persist, so XU, Y, U, UY, V^c lie in S_0 . No containment of V in S_0 is used: its budget and cut-edge count equal those of its complement. The actual cut counts are $16nD$ for XU, V , $20nD$ for U, UY , and $40nD$ for Y . The lattice estimate with coefficient 40 chooses C_B before all scanners, keeping the original support labels and the growing factor D .

For this same pair $(e, \tilde{\Omega})$, the moment estimate holds on the common real interval $|u| \leq c_0/\sqrt{\mathcal{B}}$, with $C_0 = 512e(2/g)$ and $c_0 = [32\sqrt{1+2/g}]^{-1}$. The energy, phase and trace-distance bounds in (16) are retained.²³

This geometric statement explicitly retains the source rows and clearance, the cardinality bound $|S_0| \leq C_{\text{card}} n^2$, and the literal inequalities $r_0 \leq D$, $1 \leq D$, $4D \leq m$, and $8Km \leq L$. For fixed $C_r \geq 0$ and $0 < \kappa < \mu$, the rounded source scales give these inequalities eventually; the threshold depends on the fixed exponent gap. This is not a threshold uniform over all positive exponent gaps. At the literal scales, the actual target bound $|T| \leq C_T n^2$ and the ambient rows give $|S_0| \leq |T| + nL \leq (C_T + 1)n^2$. For fixed $0 < \ell < 1$ and $0 < \kappa < \mu < 1 - \ell$, one threshold also gives $K \geq 1$. The resulting source-scale statement chooses the truncation coefficient and threshold before q , then the common budget coefficient before every physical instance. It retains the same truncated ground vector and all quantitative bounds, and assumes no compact-cardinality or literal scale-inequality conclusions.²⁴ The actual ambient rows, target size and physical cut clearance remain geometric hypotheses from the source.

The target typical-spectrum bound is $Cn(\log n)^{12}$ rather than the status bound $CnD(\log n)^{12}$. The target typical set, its positive mass and dimension, the initial prevector, the comparator estimates, and the scanner's measure-valued fields remain outside this common-status result. It does not complete the scanner data package or remove the restriction on Proposition 9.2.

²³The common physical statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.exists_truncated_statusMarginal_moment_bounds`.

²⁴The source-scale statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.exists_source_statusMarginal_moment_bounds`.

12 Target typical window

The target specialization is now stated in `TNLean/PEPS/AreaLaw/Scan/TargetTypicalWindow.lean`. Fix $q \geq 1$, $R, J \geq 0$, $\Delta > 0$ and $C_T \geq 0$. The constants $C_{\text{tr}} > 0$, $C_B > 0$ and $N \geq 2$ precede every scale, finite domain, Hamiltonian, cut and scanner. The source assumptions [Ope26] (`08-scanner.tex`, lines 21–48, 243–257 and 400–414) are a nonempty ambient target T with $|T| \leq C_T n^2$, actual ambient depth $d(x) = \text{dist}_\infty(x, T)$, at most n sites per positive ambient layer through $L = \lfloor n^{1-\ell_T} \rfloor$ for a fixed exponent $0 < \ell_T < 1$, and clearance $\text{dist}_\infty(T, Z) > 2L + 10r_0$. Here, for a set Y of sites, $\partial_\Lambda Y$ is the set of nearest-neighbour edges of the finite domain Λ with exactly one endpoint in Y , and Z consists of both sites of every edge of $\partial_\Lambda A$. The source exponent range gives $1 \leq L \leq n$; this is derived from the floor rather than assumed. Put $X = A \cap T$ and $S_0 = A \cap T_L = \{x \in A : d(x) \leq L\}$. These hypotheses yield²⁵

$$|S_0| \leq (C_T + 1)n^2, \quad X \subseteq S_0, \quad |\partial_\Lambda X| \leq 4n. \quad (17)$$

The lattice crossing-budget theorem accounts for the multiplicity of original admissible labels and the actual radius $r_0 = \lceil C_{\text{tr}}(\log n)^2 \rceil$. Thus, for this target,²⁶

$$1 \leq \mathcal{B}_X \leq C_B n (\log n)^{12}. \quad (18)$$

Only its $4n$ boundary permits using the budget factor $D = 1$; the status bound remains $CnD(\log n)^{12}$. The same actual physical pair $(e, \tilde{\Omega})$ retains (13), (16), and all-cut moment and tail estimates. At $w = n^{3/5}$ its target tail is at most n^{-100} for $n \geq N$. The canonical set of positive eigenvalues within this surprisal window [Ope26] (`07-comparators.tex`, lines 37–53) is nonempty, and its mass z and each selected eigenvalue p_i satisfy

$$1 - n^{-100} \leq z \leq 1, \quad e^{-s_X - w} \leq p_i \leq e^{-s_X + w}. \quad (19)$$

In particular $z > 0$. Here $s_X = S_{\tilde{\Omega}}(X)$ is the target entropy of $\tilde{\Omega}$, whereas the original Ω continues to center every Hamiltonian term.²⁷

This target-window statement does not construct the initial physical pre-vector, its Bell factors, the comparator estimates, the scanner’s measure-valued fields, or the complete scanner data package. It does not supply normalized typical-sector rank or entropy estimates for the later comparator construction. No auxiliary unique-gap assertion is made. The restriction on Proposition 9.2 therefore remains.

²⁵The floor bound is `TNLean.PEPS.AreaLaw.Scan.ScannerExponents.one_le_L_le`; the geometric statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.source_target_geometry`.

²⁶The lower bound is `TNLean.PEPS.AreaLaw.one_le_cutBudget`; the upper bound is part of `TNLean.PEPS.AreaLaw.Scan.CollarScan.exists_truncated_target_typical_window`.

²⁷The same-witness statement is `TNLean.PEPS.AreaLaw.Scan.CollarScan.exists_truncated_target_typical_window`.

13 Rounded weights and the marginal-moment interval

Fix $0 < \ell_s < 1$ and $0 < \kappa < \mu < 1 - \ell_s$. For the literal source scales $L = \lfloor n^{1-\ell_s} \rfloor$, $m = \lfloor n^\mu \rfloor$, $K = \lfloor L/(8m) \rfloor$ and $D = \lceil n^\kappa \rceil$, fix $\omega \in \mathbb{R}$, $C_B, A \geq 0$ and $c > 0$. The separate numerical condition

$$\omega + (1 + \kappa)/2 < 1 - \ell_s - \mu \quad (20)$$

gives one threshold, chosen before n, W, B , at which every $0 \leq W \leq n^\omega$ and $1 \leq B \leq C_B n D (\log n)^{12}$ obey $Aa \leq c/\sqrt{B}$, where $a = W/K$.²⁸ Condition (20) is not inferred from the ordinary scanner hypotheses and is not asserted for the bootstrap parameter choice.

The final collar parameters of [Ope26, Section 9, lines 839–865] have the exact gap 499879/2000000. Consequently, for every fixed $t \in \mathbb{R}$, $|ta| \leq c/\sqrt{B}$ eventually when $W = n^{1/20000}$, with the threshold uniform in all admissible budgets.²⁹ The square-root estimate retains the factor D and the sixth power of the logarithm. Applied to the already chosen common physical budget, this numerical implication leaves that budget and its truncated ground vector unchanged. Deriving the sharp norm comparisons, identifying their actual physical inputs, and completing the scanner data package remain separate obligations.

14 Elimination

The restriction disappears once the data package is constructed from the scan geometry of Lemma 9.1 and the transported states of Proposition 7.4, with Propositions 8.1 and Lemmas 2.1 and 2.3 supplying the remaining fields. Those inputs are separate items of the same formalization program.

References

- [Ope26] OpenAI. A two-dimensional area law from a global spectral gap, 2026. September 24, 2026 manuscript.

²⁸The numerical statement is `TNLean.PEPS.AreaLaw.Scan.ScannerExponents.exists_a_le_moment_radius`.

²⁹The exact final-scale statement is `TNLean.PEPS.AreaLaw.Scan.exists_final_a_le_moment_radius`.