Wolf Theorem 6.16: The Two Schwarz Orientations in the Printed Proof

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Verdict: local-correction (resolved).

1 Statement and invoked fixed-point theorem

Let $T : M_d(\mathbb{C}) \to M_d(\mathbb{C})$ be positive and trace preserving, and let T^* be its trace-pairing adjoint. Theorem 6.16 of Ref. [Wol12] assumes that T satisfies the Schwarz inequality. It concludes that the peripheral space has the weighted matrix-block form in source Equations (6.66)–(6.67), and that T acts through the block permutation and unitary conjugations in source Equation (6.68). The local source is `Notes/WolfNoteTexSource/ch06_spectral_properties.tex`, lines 1592–1624.

The proof takes a subsequential limit of powers,

$$I = \lim_{i\to\infty} T^{n_i}, \tag{1}$$

and states that I remains Schwarz because the Schwarz property is preserved under composition. It later excludes matrix transposition because T, and therefore each induced block map T_k, is Schwarz. These steps use the Schwarz inequality for T itself; see local source lines 1626–1633 and 1660–1663.

The same proof identifies the peripheral space with $\mathrm{Fix}(I)$ and invokes Theorem 6.14 of Ref. [Wol12] to obtain the weighted matrix-block structure. That theorem assumes that the trace adjoint of its trace-preserving map is Schwarz. Applied to I, the required hypothesis is

$$I^*(A^\dagger A) \succeq I^*(A^\dagger)I^*(A) \qquad \text{for every } A \in M_d(\mathbb{C}). \tag{2}$$

The local source for this hypothesis is lines 1467–1474. Closure under powers gives the Schwarz inequality for I from the corresponding inequality for T, but it does not give (2). The printed proof supplies no argument that changes the orientation.

2 Corrected proof contract

The minimal explicit correction that validates the printed proof is to assume both Schwarz inequalities:

$$T(A^\dagger A) \succeq T(A^\dagger)T(A) \qquad \text{for every } A \in M_d(\mathbb{C}), \tag{3}$$
$$T^*(A^\dagger A) \succeq T^*(A^\dagger)T^*(A) \qquad \text{for every } A \in M_d(\mathbb{C}). \tag{4}$$

The first orientation is used for the block dynamics and the exclusion of transposition. The second is stable under powers and finite-dimensional limits. Indeed,

$$(T^n)^* = (T^*)^n, \tag{5}$$
$$I^* = \lim_{i\to\infty} (T^*)^{n_i}. \tag{6}$$

Thus (4) implies (2), and Theorem 6.14 may be applied to I with its literal source hypothesis.

This correction does not claim that the conclusion of Theorem 6.16 is false under the single printed Schwarz hypothesis. A different proof might establish the required fixed-point structure. The correction records only the contract supported by the proof as written. In particular, a formal theorem following that route cannot infer that the trace-pairing adjoint of an arbitrary Schwarz map is Schwarz.

3 Formalization boundary

The source-general fixed-point theorem has the literal interface of Theorem 6.14 of Ref. [Wol12]: the map S is positive and trace preserving, and S^* satisfies the Schwarz inequality.[1]

The completed source-facing assembly for Theorem 6.16 exposes both orientations. Its recurrent-projection component passes the adjoint orientation to the fixed-point theorem, while its block-classification component retains the orientation for the original map.[2] The construction is organized by GitHub issue #40; this contract audit is GitHub issue #407.

4 Conclusion

The printed proof of Theorem 6.16 uses both Schwarz orientations but states only the one for T [Wol12, Theorem 6.16]. Adding the Schwarz inequality for T^* is a local hypothesis correction that validates the invocation of Theorem 6.14 without changing the source terminology, block notation, or proof order. Unless

[1] Formal declaration: `IsPositiveMap.exists_fixedPoints_densityBlocks_with_zero` in `QICLean/Channel/FixedPoint/WolfTheorem614`.

[2] Formal declaration: `IsPositiveMap.exists_wolfTheorem616` in `QICLean/Channel/Peripheral/StructureOfCycles`. It preserves the exact zero-extended density-block membership form and proves the output-indexed unitary action in source Equation (6.68) of Ref. [Wol12].

a separately sourced derivation removes the extra assumption, both orientations remain the source-facing formal contract.

References

[Wol12] Michael M. Wolf. Quantum channels & operations: Guided tour. Lecture notes, 2012.