Formalization-Derived Corrections to Wolf's *Quantum Channels & Operations*

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Verdict: local-correction (resolved).

Abstract

This note records discrepancies in Michael M. Wolf's *Quantum Channels & Operations: Guided Tour* that were exposed by the present Lean formalization or by the source-faithfulness analysis required for that formalization [Wol12]. It is not a general errata list. The main body gives the formalization-derived corrections, and the appendix records three separately audited errors in the local PDF transcription. Each entry states the source assertion, gives the local correction, and explains why the correction is necessary.

1 Scope

The source PDFs are stored in `Notes/WolfNotePDF/`, and the local transcription is stored in `Notes/WolfNoteTexSource/`. The corresponding formal declarations and paper-gap records are identified below. The entries distinguish three cases:

- A *source error* is a printed mathematical assertion that is false as stated.
- A *well-posedness correction* makes an implicit boundary convention explicit without changing the substantive theorem.
- A *formalization scope gap* records that the current Lean theorem is narrower than the source theorem or that an auxiliary proof is incomplete. It is not an erratum.

Only the first two cases appear in the formalization-derived main body. The appendix treats transcription errors separately.

2 The trace sign in the Lorentz-cone converse

Formalization trigger. The formal converse requires the trace to be nonnegative. Its declaration is `Matrix.IsHermitian.posSemidef_of_tra` ce_re_nonneg_of_card_sub_one_mul_trace_sq_re_le in QICLean/Analysis/MatrixTraceInequalities. The discrepancy is tracked in docs/paper-gaps/wolf_ch3_lorentz_cone_trace_sign.tex.

Printed assertion. At the end of the discussion of automorphisms of the positive cone, the first part of Chapter 3, Proposition 3.7 gives a necessary trace inequality for a positive semidefinite Hermitian matrix. Its converse is intended to characterize the associated Lorentz cone. For a Hermitian matrix $A \in M_d(\mathbb{C})$, the local PDF on book p. 66 prints

$$\operatorname{tr}(A)^2 \geq (d-1)\operatorname{tr}(A^2) \Longrightarrow A \geq 0. \tag{1}$$

This is the printed converse of Proposition 3.7 [Wol12, Proposition 3.7].

Correction. The valid converse is

$$A = A^\dagger, \qquad \operatorname{tr}(A) \geq 0, \qquad \operatorname{tr}(A)^2 \geq (d-1)\operatorname{tr}(A^2) \Longrightarrow A \geq 0. \tag{2}$$

The complex-matrix Lean statement uses real parts of the traces. For Hermitian matrices, these real parts equal the displayed real quantities.

Necessity of the correction. Set $A = -\mathbb{1}_d$. Then A is Hermitian and is not positive semidefinite, but

$$\operatorname{tr}(A)^2 = d^2, \tag{3}$$
$$(d-1)\operatorname{tr}(A^2) = d(d-1), \tag{4}$$
$$\operatorname{tr}(A)^2 \geq (d-1)\operatorname{tr}(A^2). \tag{5}$$

Thus the squared inequality describes both orientations of the Lorentz cone. The condition $\operatorname{tr}(A) \geq 0$ selects the positive sheet. The source proof also uses this sign: after bounding $\operatorname{tr}(A)$ above by a nonnegative quantity in the presence of a negative eigenvalue, it squares the bound [Wol12, Proposition 3.7]. That step requires the trace sign.

Classification. The unrestricted converse in Eq. 1 is a source error. The formal theorem proves the corrected statement in Eq. 2.

3 The missing nonvanishing condition in Kramers' theorem II

Formalization trigger. The conditional form is formalized by Matrix.IsHermitian.two_le_finrank_eigenspace_of_intertwiner_mulVec_star_ne_zero, and the invertible-intertwiner form is formalized by Matrix.IsHermitian.two_le_finrank_eigenspace_of_antisymmetric_isUnit in QICLean/Algebra/KramersDegeneracy. The same file recovers the first Kramers theorem as the unitary special case. The source error is documented in docs/paper-gaps/wolf_ch3_kramers_theorem_ii.tex.

Printed assertion. Wolf first states that if a Hermitian matrix H commutes with an anti-unitary T satisfying $T^2 = -\mathbb{1}$, then every eigenvalue of H has multiplicity at least two [Wol12]. The subsequent "Kramer's theorem II" asserts that if

$$H = H^\dagger, \tag{6}$$
$$HA = AH^T, \tag{7}$$
$$A^T = -A, \qquad A \neq 0, \tag{8}$$

then every eigenvalue of H has multiplicity at least two, "as the same proof shows" [Wol12]. The local source is `Notes/WolfNoteTexSource/ch03_positive_not_completely.tex`, lines 527–530; the matrix reduction occurs at lines 500–501.

Correction. Let $|\psi\rangle$ be a λ-eigenvector of H. Under Eq. 7, the vector $A|\overline{\psi}\rangle$ is another λ-eigenvector. Antisymmetry makes it orthogonal to $|\psi\rangle$. These facts imply degeneracy only if

$$A|\overline{\psi}\rangle \neq 0. \tag{9}$$

The correct local conclusion is therefore

$$A|\overline{\psi}\rangle \neq 0 \Longrightarrow \dim\ker(H - \lambda\mathbb{1}) \geq 2. \tag{10}$$

A global conclusion follows if A is injective. In finite dimensions, the corrected global theorem may assume that A is invertible and then conclude that every eigenvalue of H has multiplicity at least two.

Necessity of the correction. On $\mathbb{C}^3$, let

$$H = \mathrm{diag}(0, 1, 1), \tag{11}$$
$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{12}$$

Then

$$HA = AH^T = A, \tag{13}$$
$$A \neq 0, \tag{14}$$
$$Ae_1 = 0. \tag{15}$$

The eigenvalue 0 is simple, so the printed conclusion fails.

Classification. This is a source error. The first Kramers theorem remains valid because it is the case $A = V^\dagger$ of the corrected theorem, where unitarity supplies the missing nonvanishing condition [Wol12].

4 The false converse for Pauli-block truncation

Formalization trigger. The forward preservation results are formalized by `Wolf.pauliBlockDiagonalTruncation_isPositiveMap` and `Wolf.pauliBlockDiagonalTruncation_isCPMap` in `QICLean/Channel/LorentzNormalForm/PauliBlockTruncation`. The source error is documented in `docs/paper-gaps/wolf_prop2_10_block_truncation_converse.tex`.

Printed assertion. After the Bloch-ball representation in source Equation (2.39), Wolf's Proposition 2.10 writes the Pauli transfer matrix of a Hermiticity-preserving map as

$$\widehat{T} = \begin{pmatrix} \widehat{T}_{00} & r^{\mathsf{T}} \\ v & \Delta \end{pmatrix}. \tag{16}$$

It then asserts that the block truncation

$$\widehat{T}' = \widehat{T}_{00} \oplus \Delta \tag{17}$$

is positive, respectively completely positive, if and only if $\widehat{T}$ is [Wol12, Proposition 2.10]. The local source is `Notes/WolfNoteTexSource/ch02_representations.tex`, lines 984–990.

Correction. Define the reduction map

$$\Theta(X) = \operatorname{tr}(X)\mathbb{1} - X \tag{18}$$

and the diagonal Pauli-coordinate matrix

$$D = \operatorname{diag}(1, -1, -1, -1). \tag{19}$$

Then the truncation satisfies

$$T' = \frac{T + \Theta \circ T \circ \Theta}{2}, \tag{20}$$

$$\widehat{T}' = \frac{\widehat{T} + D\widehat{T}D}{2}, \tag{21}$$

$$\widehat{T}' = \widehat{T}_{00} \oplus \Delta. \tag{22}$$

The valid implications are

$$T \text{ positive} \Longrightarrow T' \text{ positive}, \tag{23}$$

$$T \text{ completely positive} \Longrightarrow T' \text{ completely positive}. \tag{24}$$

These are the directions proved at local source lines 992–998 [Wol12, Proposition 2.10] and formalized by the declarations above.

Necessity of the correction. Consider

$$T(X) = \mathrm{tr}(X)\,\mathrm{diag}(3/2, -1/2). \tag{25}$$

This map preserves Hermiticity but is not positive because

$$T(\mathbb{1}/2) = \mathrm{diag}(3/2, -1/2) \ngeq 0. \tag{26}$$

Its Pauli-block truncation is

$$T'(X) = \mathrm{tr}(X)\frac{\mathbb{1}}{2}, \tag{27}$$

which is the completely depolarizing channel and is completely positive. Hence both printed converses fail.

Classification. This is a source error. The proof of Proposition 2.10 supports Eqs. 23 and 24, not the printed equivalences [Wol12].

5 The inverse coefficient in the proof of Theorem 2.3

Formalization trigger. The formal proof defines Wolf's normalized auxiliary matrix by `stinespringW` and proves injectivity of the assignment from a supplied Stinespring matrix by `stinespringW_injective`, both in `QICLean/Channel/OrderedCP`.

Printed assertion. In the proof of Theorem 2.3, Wolf uses the normalized vector

$$|\Omega\rangle = d'^{-1/2} \sum_a |a, a\rangle \tag{28}$$

and defines

$$W_i = (\mathbb{1}_{r_i} \otimes \langle\Omega|)(V_i \otimes \mathbb{1}_{d'}). \tag{29}$$

The accompanying footnote states that $V_i \mapsto W_i$ is injective but prints d'^2 as the coefficient in the inverse [Wol12, Theorem 2.3]. The same coefficient occurs in `Notes/WolfNoteTexSource/ch02_representations.tex`, lines 392–393, and in the PDF on book p. 39.

Correction. The coordinate relation is

$$(W_i)_{j,(k,a)} = d'^{-1/2}(V_i)_{(a,j),k}. \tag{30}$$

Its coordinate inverse therefore multiplies by $\sqrt{d'}$. With the canonical tensor-factor identifications used in the source, the inverse is

$$V_i = d'(W_i \otimes \mathbb{1}_{d'})(\mathbb{1}_d \otimes |\Omega\rangle). \tag{31}$$

Necessity of the correction. The second occurrence of $|\Omega\rangle$ contributes another factor $d'^{-1/2}$. Consequently,

$$(W_i \otimes \mathbb{1}_{d'})(\mathbb{1}_d \otimes |\Omega\rangle) = \frac{1}{d'} V_i. \tag{32}$$

The coefficient in Eq. 31 must therefore be d', not d'^2. Theorem 2.3 is unaffected: the formal injectivity proof uses Eq. 30 and cancels only the nonzero factor $d'^{-1/2}$.

Classification. This is a normalization typo in a source footnote. It changes neither the statement nor the proof route of Theorem 2.3 [Wol12].

6 The nonempty-family boundary in the Radon–Nikodym theorem

Formalization trigger. The source-facing theorem `IsKrausCP.radon_nikodym_of_stinespring` assumes that the finite index type is nonempty. It uses Wolf's rectangular Heisenberg dimensions $T_i, T : \mathcal{M}_{d'} \to \mathcal{M}_d$ and $V : \mathbb{C}^d \to \mathbb{C}^{d'} \otimes \mathbb{C}^r$. The boundary is documented in `docs/paper-gaps/wolf_radon_nikodym_nonempty_family.tex`.

Printed assertion. Wolf's Theorem 2.4 is the instrument-valued corollary of the preceding comparison theorem for ordered completely positive maps. It fixes a Stinespring representation of the total map and represents each component by a positive effect on the same dilation space. The local source is `Notes/WolfNoteTexSource/ch02_representations.tex`, lines 399–407. It states:

> "Let $\{T_i\}$ be a set of completely positive linear maps such that $\sum_i T_i = T$ ... Then there is a set of positive operators $P_i \in M_r$ which sum up to I_r such that $T_i(A) = V^\dagger(A \otimes P_i)V$."

This is Theorem 2.4 of Ref. [Wol12].

Formal statement. Let ι be a nonempty finite index type. The formal theorem assumes completely positive maps $T_i, T : \mathcal{M}_{d'} \to \mathcal{M}_d$ satisfying

$$\sum_{i \in \iota} T_i = T \tag{33}$$

and a supplied map $V : \mathbb{C}^d \to \mathbb{C}^{d'} \otimes \mathbb{C}^r$ satisfying

$$T(A) = V^\dagger(A \otimes I_r)V. \tag{34}$$

It concludes that positive semidefinite operators $P_i \in \mathcal{M}_r$ exist such that

$$\sum_{i \in \iota} P_i = I_r, \tag{35}$$

$$T_i(A) = V^\dagger(A \otimes P_i)V. \tag{36}$$

The declaration `IsCPMap.radon_nikodym_of_stinespring` remains the square-algebra specialization.

Formal proof path. Wolf presents this result as a simple corollary of the ordered-map comparison theorem and gives no separate proof [Wol12, Theorem 2.4]. The formal proof combines Kraus families for the components into an outcome-labelled Stinespring representation $\widehat{V}$ of T, then applies the preceding theorem to $T \leq T$ and the representations $\widehat{V}$ and V. It obtains a contraction C satisfying

$$\widehat{V} = (I_{d'} \otimes C)V, \tag{37}$$

$$C^\dagger C \leq I_r. \tag{38}$$

This contraction need not be an isometry.

Let C_i retain the rows of C labelled by i, and define

$$E_i = C_i^\dagger C_i. \tag{39}$$

The operators E_i are positive, represent T_i, and satisfy

$$\sum_{i \in \iota} E_i = C^\dagger C. \tag{40}$$

The residual operator

$$R = I_r - C^\dagger C \tag{41}$$

is positive and satisfies

$$V^\dagger (A \otimes R) V = 0. \tag{42}$$

Choose $i_0 \in \iota$ and set

$$P_{i_0} = E_{i_0} + R, \tag{43}$$

$$P_i = E_i \qquad (i \neq i_0). \tag{44}$$

This step gives Eq. 35 for an arbitrary nonminimal supplied dilation and is where nonemptiness is used. If $d' = 0$, the formal proof instead assigns I_r to a chosen outcome and zero to every other outcome. The square-algebra declaration specializes the rectangular theorem directly.

Necessity of the boundary. If ι is empty, then Eq. 33 forces $T = 0$. The zero Stinespring matrix $V = 0$ satisfies Eq. 34, but the printed conclusion would require

$$0 = \sum_{i \in \iota} P_i = I_r. \tag{45}$$

This is false when the dilation space has nonzero dimension. A nonempty finite outcome family is also the substantive case for a quantum instrument.

Classification. This is a local well-posedness correction, not a claim that the nonempty-family Radon–Nikodym theorem is false. The word "set" and the summation notation in the source leave the empty-family boundary implicit [Wol12]; Lean must state it explicitly.

7 Suppressed zero multiplicities in the two-pure-state spectrum

Formalization trigger. The theorem `Projectivization.charpoly_add_pureStateMatrix_of_two_le` retains the full characteristic polynomial of the sum of two normalized pure-state matrices. The correction is documented in `docs/paper-gaps/wolf_ch01_two_pure_state_zero_multiplicities.tex`.

Printed assertion. In the spectrum-preserver argument following Wigner's theorem, Wolf states that the two potentially nonzero eigenvalues of the sum of two Hermitian rank-one projections P, Q are

$$1 \pm \sqrt{\operatorname{tr}(PQ)}. \tag{46}$$

These values are the roots of

$$(X-1)^2 - \operatorname{tr}(PQ). \tag{47}$$

The statement appears in Chapter 1 of Ref. [Wol12].

Correction. For $d \geq 2$, the multiplicity-aware identity is

$$\chi_{P+Q}(X) = X^{d-2}((X-1)^2 - \operatorname{tr}(PQ)). \tag{48}$$

The factor X^{d-2} supplies the guaranteed additional zero multiplicity. If $P = Q$, then

$$(X-1)^2 - \operatorname{tr}(PQ) = X(X-2), \tag{49}$$

so the total zero multiplicity is $d-1$. In dimension one,

$$\chi_{P+Q}(X) = X - 2. \tag{50}$$

In dimension zero, no projective pure states exist.

Classification. This is a local correction to the spectrum shorthand in Eq. 46, not a claim that the intended spectrum-preserver argument is false. The two displayed values retain the transition-probability information used in that argument [Wol12]; the characteristic polynomial in Eq. 48 records all multiplicities.

8 The rotation sign in the spinor exponential formula

Formalization trigger. The rotation exponential is formalized by `Wolf.spinorMatrix_rotationExpSL2` in `QICLean/Channel/LorentzNormalForm/SpinorExponential`. The discrepancy is documented in `docs/paper-gaps/wolf_ch2_spinor_rotation_sign.tex`.

Printed assertion. Source Equation (2.44), at `Notes/WolfNoteTexSource/ch02_representations.tex`, lines 1070–1077, defines

$$R_i = \sum_{j,k=1}^{3} \varepsilon_{ijk} |k\rangle\langle j| \tag{51}$$

and prints the correspondence

$$e^{-i\vec{n}\cdot\vec{\sigma}/2} \longmapsto e^{-\vec{n}\cdot\vec{R}}. \tag{52}$$

This is the rotation part of Equation (2.44) in Ref. [Wol12].

Correction. With the standard Pauli matrices and the column-vector convention used in the notes, the correct correspondence is

$$e^{-i\vec{n}\cdot\vec{\sigma}/2} \longmapsto e^{+\vec{n}\cdot\vec{R}}. \tag{53}$$

Equivalently, the printed Lorentz exponential corresponds to a spinor with the opposite sign in its exponent.

Necessity of the correction. For $\vec{n} = (0, 0, \theta)$, direct Pauli conjugation gives

$$e^{-i\theta\sigma_3/2}\sigma_1 e^{i\theta\sigma_3/2} = \cos\theta\,\sigma_1 + \sin\theta\,\sigma_2. \tag{54}$$

The generator in Eq. 51 satisfies

$$R_3|1\rangle = |2\rangle. \tag{55}$$

Therefore Eq. 54 is the action of $e^{+\theta R_3}$, whereas $e^{-\theta R_3}$ rotates toward the negative second coordinate.

Classification. This is a source sign error. The formal theorem follows the actual congruence action. The boost formula in the same source equation is unchanged [Wol12, Equation (2.44)].

9 The two boundary errors in Theorem 6.3

Formalization trigger. The declarations `exists_wolfTheorem63_of_irreducible_positive` and `exists_wolfTheorem63_of_irreducible_positive_of_ne_zero` formalize Wolf's Theorem 6.3 for arbitrary positive maps. The declarations `lowerCollatzWielandtValue`, `upperCollatzWielandtValue`, and `lowerCollatzWielandtGlobal_eq_upperCollatzWielandtGlobal` formalize the corrected pointwise and global quantities. Both corrections are documented in `docs/paper-gaps/wolf_thm6_3_positive_map_cp_scope.tex`.

The global upper quantity. After defining the pointwise lower and upper functionals $r(X)$ and $\widetilde{r}(X)$, local source lines 617–619 print

$$r = \sup_{X \geq 0} r(X), \tag{56}$$

$$\widetilde{r} = \sup_{X \geq 0} \widetilde{r}(X), \tag{57}$$

$$r \geq \widetilde{r}. \tag{58}$$

These formulas occur in the proof of Theorem 6.3 of Ref. [Wol12]. The corrected Collatz–Wielandt quantities are

$$r = \sup_{\substack{X \geq 0 \\ X \neq 0}} r(X), \tag{59}$$

$$\widetilde{r} = \inf_{\substack{X \geq 0 \\ X \neq 0}} \widetilde{r}(X). \tag{60}$$

For an irreducible positive map, both extrema are attained at the same positive-definite eigenvector and are equal. Thus the second supremum and the reversed inequality in Eq. 58 are source errors. Equality is not a pointwise statement for arbitrary X. In the extended-real formulation,

$$r(0) = +\infty, \tag{61}$$

$$\widetilde{r}(0) = -\infty, \tag{62}$$

whereas every nonzero $X \geq 0$ satisfies

$$r(X) \leq \widetilde{r}(X). \tag{63}$$

Equality holds at positive eigenvectors, as used at local source lines 649–651 [Wol12].

The one-dimensional zero map. Theorem 6.3 assumes only that $T : M_D(\mathbb{C}) \to M_D(\mathbb{C})$ is positive and irreducible, but its second item and proof require a strictly positive Perron value [Wol12, Theorem 6.3]. On $M_1(\mathbb{C})$, the zero map is positive and irreducible because 0 and $\mathbb{1}$ are the only orthogonal projections. Its spectral radius is

$$r(T) = 0. \tag{64}$$

The theorem without a nonzero-map hypothesis must therefore allow $r = 0$ while retaining a positive-definite eigenvector $X > 0$. The printed conclusion $r > 0$ becomes valid after adding $T \neq 0$. When $D > 1$, irreducibility itself excludes the zero map.

Classification. These are two local source errors exposed by the positive-map formalization. They do not alter the proof for a nonzero irreducible map once the second global extremum is interpreted as the infimum in Eq. 60.

10 The missing order hypothesis in the unitary comparison

Formalization trigger. The correction and the source proof route are documented in `docs/paper-gaps/wolf_ch5_unitary_comparison_missing_order.tex`. The cross-matrix Weyl comparison is formalized by `LinearMap.IsSymmetric.eigenvalues_le_of_sub_isPositive`, and the corrected proposition is formalized by `Matrix.IsHermitian.exists_unitary_cfc_le_cfc_of_le`.

Printed assertion and correction. Immediately after stating Weyl monotonicity for Hermitian matrices satisfying $A \leq B$, Proposition 5.3 omits this order hypothesis [Wol12, Proposition 5.3]. The corrected statement, with the unitary chosen before the interval and the function, is

$$\begin{aligned} A \leq B \Longrightarrow \exists U \in M_d(\mathbb{C}) \ \Big[& U^\dagger U = UU^\dagger = \mathbb{1} \qquad (65)\\ & \wedge \forall I \subseteq \mathbb{R}\ \forall f : I \to \mathbb{R},\\ & \big(I \text{ is an interval} \wedge \operatorname{spec}(A) \cup \operatorname{spec}(B) \subseteq I \wedge f \text{ is non-decreasing on } I\big)\\ & \qquad \Longrightarrow f(A) \leq U f(B) U^\dagger \Big]. \qquad (66) \end{aligned}$$

This is the corrected form of source Equation (5.57) [Wol12, Equation (5.57)].

In dimension one, take

$$A = [1], \tag{67}$$
$$B = [0], \tag{68}$$
$$I = [0, 1], \tag{69}$$
$$f(x) = x. \tag{70}$$

Every unitary is a phase, so the unqualified printed conclusion becomes

$$1 \leq 0, \tag{71}$$

which is false. The hypothesis $A \leq B$ is indispensable.

Source proof route. Under $A \leq B$, source Equation (5.56) gives

$$\lambda_j^{\downarrow}(A) \leq \lambda_j^{\downarrow}(B) \tag{72}$$

for every j, because the smallest eigenvalue of $B - A$ is nonnegative [Wol12, Equation (5.56)]. Monotonicity of f gives

$$f(\lambda_j^{\downarrow}(A)) \leq f(\lambda_j^{\downarrow}(B)). \tag{73}$$

Choose unitaries V_A, V_B whose columns are eigenvectors in decreasing eigenvalue order, and define

$$U = V_A V_B^{\dagger}. \tag{74}$$

Transporting Eq. 73 through these eigenbases yields

$$f(A) \leq U f(B) U^{\dagger}. \tag{75}$$

The unitary in Eq. 74 depends only on the two ordered eigenbases, so it works for every admissible f, as required by the quantifier order in Proposition 5.3 [Wol12]. The formal cross-matrix eigenvalue comparison uses the intersection of the span of the first $j + 1$ ordered eigenvectors of A with the span of the last $d - j$ ordered eigenvectors of B; it does not assume a pointwise comparison.

Classification. This is a source error. The printed theorem is false, whereas the corrected statement in Eq. 66 follows from the preceding source argument once $A \leq B$ is retained.

11 The subunital logarithm clause in Corollary 5.2

Formalization trigger. The logarithmic Jensen declarations `posMap_log_concave_jensen` and `IsPositiveMap.log_concave_jensen`, in `QICLean/Analysis/LiebConcavity` and `QICLean/Channel/Schwarz/OperatorConvexity`, assume that the positive map is unital. The discrepancy is documented in `docs/paper-gaps/wolf_ch5_operator_jensen_lieb.tex`.

Printed assertion. Chapter 5, Corollary 5.2 places three conclusions under the common hypothesis that T is positive and $T(\mathbb{1}) \leq \mathbb{1}$. Its third conclusion is

$$T(\log A) \leq \log(T(A)), \qquad A > 0. \tag{76}$$

The local source is `Notes/WolfNoteTexSource/ch05_schwarz_inequalities.tex`, lines 761–769; see Corollary 5.2 of Ref. [Wol12].

Correction. The proved logarithmic statement requires unitality:

$$T(\mathbb{1}) = \mathbb{1}, \qquad A > 0 \Longrightarrow T(\log A) \leq \log(T(A)). \tag{77}$$

The two real-power clauses in the same corollary remain valid under the subunital hypothesis printed by Wolf [Wol12, Corollary 5.2]. The live Jensen declarations use that hypothesis for the power clauses, so the logarithmic correction does not alter them.

Necessity of the correction. Fix $0 < c < 1$ and consider the one-dimensional positive map

$$T_c(x) = cx. \tag{78}$$

For $A = 1$,

$$T_c(1) = c \leq 1, \tag{79}$$
$$T_c(\log 1) = 0, \tag{80}$$
$$\log(T_c(1)) = \log c < 0. \tag{81}$$

Hence

$$T_c(\log 1) > \log(T_c(1)), \tag{82}$$

so the printed clause fails even for a strictly positive scalar map. Wolf's preceding positive-map monotonicity theorem also requires a function on an interval containing zero with $f(0) \geq 0$, whereas the logarithm is unbounded below at zero [Wol12].

Classification. This is a source error in the common hypothesis of Corollary 5.2. The formal theorem proves Eq. 77 rather than the false subunital target.

12 Formalization gaps that are not source corrections

The following Wolf-specific notes record formalization scope or dependency boundaries rather than source errata:

- `wolf_prop28_square_case.tex` records that the current normal-form theorem is the square-dimensional special case of Wolf's rectangular statement [Wol12].
- `wolf_lemma_3_1_top_index_scope.tex`, `wolf_prop_3_2_top_index_scope.tex`, and `wolf_t_eta_top_index_scope.tex` record endpoint Schmidt-rank cases that were formalization scope gaps and are now resolved.

- `wolf_reduction_criterion_schmidt_premise.tex` records a formalization dependency on the Schmidt-number premise that is now resolved.
- `wolf_ex3_1_choi_positivity_subcase_scope.tex` records that the development proves selected parameter families rather than the full source range.
- `wolf_cor67_maximal_support_restriction.tex` records a resolved formalization boundary rather than a false printed theorem.

13 Conclusion

The formalization-derived record identifies source errors in Sections 2, 3, 4, 8, 9, 10, and 11. It also identifies one normalization typo in Section 5, one explicit boundary convention in Section 6, and one local correction to a spectrum shorthand in Section 7. These are the corrections exposed by the Lean development and its source-faithfulness audits of Ref. [Wol12].

A Corrected errors in the local LaTeX transcription

This appendix records errors found by comparing the source PDFs with the local transcription. They are not claims that a Lean proof forced a correction to Wolf's statements. Each error was mathematically material and has been corrected in the local TeX source.

A.1 The Choi marginal: τ_B versus $\operatorname{tr}_B \tau$

Source context. Immediately after deriving the coordinate form of the Choi–Jamiolkowski correspondence in source Equation (2.4), Wolf describes the channel–state dictionary [Wol12, Equation (2.4)]. For a channel $T : M_d \to M_{d'}$, the first tensor factor is the output factor A, and the second is the untouched input or reference factor B. The PDF on book p. 34 states:

> "If T is a quantum channel in the Schrödinger picture, then τ is a density operator with reduced density matrix $\tau_B = \mathbb{1}/d$."

Thus τ_B denotes $\operatorname{tr}_A \tau$.

Transcription and correction. A previous local transcription at `ch02_representations.tex:113--114` replaced this marginal by $\operatorname{tr}_B \tau = \mathbb{1}/d$. The source-faithful identity is

$$\tau_B = \operatorname{tr}_A \tau = \frac{\mathbb{1}_d}{d}. \tag{83}$$

Reason. Trace preservation is equivalent to Eq. 83. The other marginal is

$$\operatorname{tr}_B \tau = \frac{T(\mathbb{1})}{d}. \tag{84}$$

It is maximally mixed precisely when T is unital. The former transcription therefore changed a trace-preserving condition into a stronger condition that is generally false.

A.2 The conjugation in the proof of Kramer's theorem

Source context. Chapter 3 proves Kramer's theorem for a Hermitian matrix H commuting with an anti-unitary symmetry $T = \Gamma V$, where $V^T = -V$ [Wol12]. After showing that $V^\dagger|\overline{\psi}\rangle$ is another eigenvector, source Equation (3.29) proves orthogonality through

$$\langle\psi|V^\dagger|\overline{\psi}\rangle = -\langle\psi|\overline{V}|\overline{\psi}\rangle, \tag{85}$$

$$\langle\psi|\overline{V}|\overline{\psi}\rangle = \overline{\langle\overline{\psi}|V|\psi\rangle}. \tag{86}$$

This is Equation (3.29) in Ref. [Wol12]. The final conjugate is then identified with $\langle\psi|V^\dagger|\overline{\psi}\rangle$, so the scalar equals its own negative.

Transcription and correction. A previous local TeX version at `ch03_positive_not_completely.tex:515` omitted the outer conjugation in the last term. The corrected term is

$$-\overline{\langle\overline{\psi}|V|\psi\rangle}. \tag{87}$$

Reason. The componentwise identity in Eq. 86 shows that the conjugation is necessary. Without it, the equality is false in general, and the final self-cancellation does not follow.

A.3 The order of the conjugated factors in Example 5.3

Source context. Chapter 5 considers the adjoint of the map

$$T(X) = \frac{1}{2}\left(X^T + \frac{1}{2}\operatorname{tr}(X)\mathbb{1}\right) \tag{88}$$

as an example of a Schwarz map that need not be 2-positive [Wol12, Example 5.3]. For traceless A, source Equation (5.10) subtracts

$$\frac{1}{4}\,\overline{A}A^T \tag{89}$$

from $T^*(A^\dagger A)$ [Wol12, Equation (5.10)].

Transcription and correction. A previous local TeX version at `ch05_schwarz_inequalities.tex:373,377` wrote $A\overline{A}^T$. It has been corrected to Eq. 89.

Reason. For traceless A,

$$T^*(A) = \frac{1}{2} A^T, \tag{90}$$

$$T^*(A^\dagger) T^*(A) = \frac{1}{4} (A^\dagger)^T A^T, \tag{91}$$

$$T^*(A^\dagger) T^*(A) = \frac{1}{4} \overline{A} A^T. \tag{92}$$

The transcribed matrix $A\overline{A}^T = AA^\dagger$ is generally different. Both matrices are positive semidefinite, so the final positivity conclusion in the example survives.

References

[Wol12] Michael M. Wolf. Quantum channels & operations: Guided tour. Lecture notes, 2012.