

Policy for Paper-Gap Notes

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1 Purpose

A paper-gap note records a mathematical discrepancy between a cited source and the statement, proof, or route used in the formal development. The source itself should remain intact. If a proof appears to require a hypothesis absent from the article, a different scalar, a different normalization, or a replacement theorem from the formal library, explain the change in a separate note under `docs/paper-gaps/`.

The note is a mathematical document first. It should be readable by a mathematician or mathematical physicist who has not read the corresponding issue or pull request. Traceability data such as formal declaration names, source-file paths, GitHub issues, and line ranges support the exposition, usually in footnotes; they do not govern it.

2 When a note is needed

A note is needed when the formal statement is not literally the cited statement. A common case is that the paper claims

$$\mathcal{H} \implies C. \tag{1}$$

The formal proof may instead establish only

$$\mathcal{H} \wedge \mathcal{K} \implies C. \tag{2}$$

The condition $\mathcal{K}$ must then be accounted for. Either it follows from $\mathcal{H}$, it is a harmless local correction, or (2) is a restricted theorem relative to (1) and must be marked as such.

A note is also needed when the proof is mathematically different even though the final conclusion is the same. For example, a source proof might identify a coefficient from the following finite power-sum identity, assumed for $1 \leq N \leq m$:

$$\sum_{q=1}^{r_a} \mu_q^N = \sum_{q=1}^{r_b} \nu_q^N. \tag{3}$$

The formal route might instead use convergence of a sequence. These are different mechanisms. The note must state which hypotheses each mechanism uses and show the intermediate equalities required by the replacement argument.

3 Mathematical presentation

Define every symbol before its first use. State the cited assertion in the special case under discussion, state the formal or corrected assertion, and compare their hypotheses and conclusions. Use precise, concise, and mathematically direct prose. Long quotations are usually unnecessary; a specific citation and the relevant formulas are more useful.

Every substantive displayed equation must use `\begin{align}` and `\end{align}`. Use multiple aligned lines when a derivation depends on intermediate equalities. An `aligned` structure may occur inside `align` only when a nested multi-line expression is mathematically necessary. Do not compress a proof gap into one implication when the missing step occurs within a calculation.

Every substantive displayed equation must have a stable semantic label of the form `\label{eq:<note-prefix>_<content>}`. The label must describe the mathematical content, not the display’s position or current number. When a display contains several independently discussed lines, label each such line. For example, a comparison argument may use

$$V^{(N)}(A) = c_N V^{(N)}(B), \quad (4)$$

$$\langle V^{(N)}(B_k) | V^{(N)}(A_j) \rangle \xrightarrow{N \rightarrow \infty} 0, \quad (5)$$

$$\dim \text{span} \left(\{V^{(N)}(A_j)\}_j \cup \{V^{(N)}(B_k)\} \right) = r_A + 1. \quad (6)$$

The prose must identify the nonzero coefficient forced to vanish and cite the step that produces the contradiction, such as (5) or (6).

Refer to a displayed equation in the local note through `\ref`. Write `(\ref{eq:<label>})` or a grammatical equivalent. Do not write “the equation above”, “the preceding display”, a raw local equation number, or an unlinked source equation number. A source-paper equation is not a local cross-reference: name the source, identify its equation, and cite it with an existing bibliography key, for example Equation (5.2) of Ref. [Wol12]. Every mathematical claim attributed to a paper or other literature source must likewise carry a `\cite{...}` citation using a key already present in `references.bib`. Do not invent citation keys.

Do not use `$$...$$`, `\[...\]`, `equation`, `equation*`, `align*`, `gather`, `gather*`, `multline`, or `multline*` for equations in paper-gap notes. In particular, an equation later discussed in prose may not appear in an unlabelled display. The only exceptions are non-equational material generated by a document package, such as a diagram or table, and a short formula retained in inline math because no display is warranted. These exceptions do not permit an unlabelled displayed equation.

Treat formulas as parts of sentences and punctuate them accordingly. Short definitions may remain inline when that placement makes the sentence clearer.

4 Formal traceability

Write formal identifiers with `\leanid` and place them in footnotes unless the formal declaration itself is the mathematical object under discussion. For example, the prose may say that the formal theorem is the same-dimensional gauge-phase component of a source lemma, while a footnote identifies the declaration.¹ Do not put formal identifiers in displayed equations merely as traceability data.

Explain a replacement theorem in ordinary mathematics before identifying its formal declaration. Verify its hypotheses in the notation of the note. A formal identifier or source-file link does not replace either step.

5 Shared commands

All notes in this directory must load `docs/paper-gaps/command.tex`. Shared notation belongs there. In particular, use the shared commands `\tr` and `\leanid` to produce $\text{tr}(X)$ and `someDeclaration`; do not redefine them in an individual note. This rule keeps the documents visually consistent and prevents contradictory local conventions.

6 Naming

A note's file name has the form `<key>_<topic>.tex`. The key names the source the note examines: the authors' initials followed by a two-digit year (`cpsv16`, `pgvwc07`, `spwc10`), or an established short name for a book or review (`wolf`, `rmp`, `mpu`). A note that audits the formal development itself, with no single external source, takes a reserved key for the project itself, namely `qiclean`. The registry of keys, with the source named by each key, is maintained in the repository configuration `texra-blueprint.toml`; the index generator rejects an unregistered key.

The topic part states the mathematical subject, not the history of the note. A file name is permanent once another document cites it. Version suffixes are not used because a note is revised in place and its history is preserved by the repository.

Blueprint prose cites a note like any other source, with `\cite{gap:<slug>}`; the bibliography entry carries the published URL of the note, whether the note lives in this repository or another one. Lean docstrings and comments cite a note owned by this repository by its repository path `docs/paper-gaps/<slug>.tex`. A note owned by another repository has no source file here, so they cite its published URL.

¹Formal declaration: `Model.correctedStatement_of_extra_hypothesis` in `Project/Path/To/File`.

7 Classification

The conclusion of a note must give a verdict. The available classifications are a notational clarification, a local correction, a scope restriction, an unfaithful theorem, a source claim that is false as printed, and an open gap. The classification follows the mathematics, not the amount of formal work required.

Each note declares its classification and status machine-readably with `\gapnote{<kind>}{<status>}` directly after `\maketitle`. The kind is one of `clarification`, `local-correction`, `scope-restriction`, `unfaithful`, `false-source`, and `open-gap`. The status is one of `open`, `wip`, `resolved`, and `historical`. The status `wip` marks a gap whose elimination is actively underway and still counts as live. Severity is derived from the kind and is never stated separately. A resolved note keeps its classification and changes only its status; the published index lists live notes first and dims resolved notes.

A local correction records a small adjustment whose corrected statement is still available in the cited argument. A scope restriction records a theorem proved only in a special case of the source theorem. An unfaithful theorem relies on a hypothesis or intermediate result that the cited source does not provide. Such a theorem must carry an in-source marker pointing to the paper-gap note. A source claim is false as printed when a counterexample satisfies the stated source hypotheses and contradicts its conclusion. This classification concerns the cited assertion itself rather than a mismatch introduced by the formal statement or proof.

8 Revision

A note may begin provisionally, but revisions must keep it as a coherent account of the present mathematical understanding rather than a chronological log. If later work proves that $\mathcal{K}$ follows from $\mathcal{H}$, rewrite the note to show that derivation. If later work confirms that the extra hypothesis is unavoidable, state the restricted theorem explicitly. Preserve stable equation labels when their mathematical content is unchanged, so references from other documents remain valid.

References

- [Wol12] Michael M. Wolf. Quantum channels & operations: Guided tour. Lecture notes, 2012.