

Literal Coherent Measures and the Remaining Scanner Construction

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Verdict: clarification (resolved).

1 The coherent representation

The coherent measures in the transport estimate of Ref. [Ope26, Proposition 7.4] are measures on the one-copy unit sphere.¹ For a positive matrix ρ , the measure is the image of Haar measure with density $D_k \operatorname{Tr}(\rho P_{Ue_a,k})$ under $U \mapsto Ue_a$. For a terminal leaf, the Fourier average has density

$$d_j(u, U) = m_{1/4}(u) D_k \operatorname{Re} \operatorname{Tr}(\sigma_{j,u} P_{Ue_a,k}). \quad (1)$$

This uses the actual terminal state. History and choice weights remain outside this measure, and its normalized-state mass is $1/2$, not one.

2 The construction and its hypotheses

Both measures are defined by weighted pushforward. Their real-integral identities require positivity of the density; the transported-state identity uses only admissibility, a nonnegative regulator parameter, and cross-band commutation on the symmetric subspace. Positivity holds at every real clamped interpolation parameter. Trace one additionally needs a nonzero supplied vector and nonzero terminal weight. At zero weight, the accepted normalization of the derivative map gives zero state and zero measure. The source itself introduces the terminal states on the open interpolation interval; the inactive boundary assertion is about this accepted extension, not an extra assertion attributed to the paper.

¹Source: `06-transport.tex`, lines 364–401, displays `transport:coherent-measure` and `transport:states`.

Absolute integrability of (1) is proved by bounding a continuous function on the product of the replica-space unitary group and the one-copy unitary group. Both groups are needed. The resulting bound may depend on the fixed interpolation parameter, replica count, and supplied vector. No continuity in the interpolation parameter follows from this argument.

3 Scope of the representation

The weighted pushforwards establish the coherent and Fourier integral representations, their finiteness, the normalized mass on active leaves, and the zero-state identities under the hypotheses above.²

The representation theorem alone does not construct physical scanner data or a comparator. Specialization to fill and charge data, identification of entropy and energy symbols, the physical supplied vectors, and the remaining parameter-continuity and endpoint norm estimates are separate steps. In particular, including zero replicas for an explicitly supplied nonzero scalar vector says nothing about a physical ground-state energy identity at zero replicas. The note records no defect in the source mathematics.

References

- [Ope26] OpenAI. A two-dimensional area law from a global spectral gap, 2026. September 24, 2026; source revision `adc7f1241b42e322a6451854ab7e4b4c146bf78a`.

²Formal modules: `QICLean/Representation/ReplicaTransport/CoherentMeasure.lean` and `QICLean/Representation/ReplicaTransport/LeafMeasure.lean`.