Global and Per-Sector Unit-Weight Witnesses in the CPSV16 BNT Argument

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Verdict: scope-restriction (resolved).

1 Closure record

Cirac–Pérez-García–Schuch–Verstraete normalize the raw coefficients in the basis-of-normal-tensors expansion by one global condition. Section II.C, line 246 of Ref. [CPGSV16] requires $|\mu_{j,q}| \leq 1$ for every copy weight and $|\mu_{j_*,q_*}| = 1$ for at least one copy. The paper-level canonical form does not require every BNT sector to contain a unit-modulus copy. One sector may contain only strictly subunit copy weights while another sector contains the global unit witness. The formal predicate `MPSTensor.IsBNTCanonicalForm` records exactly this global condition through the field `MPSTensor.IsBNTCanonicalForm.weight_unit_exists`.

Earlier full-basis matching and global-gauge theorems assumed the stronger per-sector condition $\forall j,\ \exists q,\ |\mu_{j,q}| = 1$. That condition entered through an asymptotic matching argument that projected onto individual sectors using the Cesàro non-decay lemma. The restriction has been eliminated.

For sector j, define the length-N coefficient sum by

$$c_N^{(j)} = \sum_q \mu_{j,q}^N. \tag{1}$$

The exact linear-independence matcher works at one fixed sufficiently large length. Its equal-case declaration is `MPSTensor.exists_block_match_exact`, and its proportional-case declaration is `MPSTensor.exists_block_match_exact_of_eventuallyProportional`. The theorem `MPSTensor.IsBNTCanonicalForm.coeff_not_eventually_zero` shows that $c_N^{(j)}$ is nonzero for infinitely many N whenever the copy weights are nonzero; it requires no modulus assumption. The matching and global-gauge declarations therefore use exactly the source hypothesis set.

The unitary equal-case assembly in Corollary A.6 of Ref. [CPGSV16], identified in the source by `thm:Fundamental-CFII` at `Papers/1606.00608/MPDO-22-12-17-2.tex:1197`, is complete under the same hypotheses. Its formal declarations are `MPSTensor.ft_sector_bnt_equal_unitary_global_gauge_witne`

sses and MPSTensor.fundamentalTheorem_equal_canonicalForm_unitary. Neither declaration assumes a per-sector unit weight.

The faithful structural normalization remains the global unit-weight field MPSTensor.IsBNTCanonicalForm.weight_unit_exists in TNLean/MPS/FundamentalTheorem/SectorBNT/Basic.lean. The interaction between this convention and the scale fixed by the MPDO renormalization equations is treated separately in docs/paper-gaps/cpsv16_unit_weight_rfp_scale_tension.tex.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.