The Accumulated Error in the Pasting From-H-to-G Lemma

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1 The assertion

This note concerns the from-H-to-G lemma in the pasting section of [JNV$^+$20].[1] Throughout this note, k is the number of pasting, or sampling, stages; m is the number of variables in each slice; d is the individual-degree bound; and q is the field size. The scalars γ and ζ are, respectively, the consistency and self-consistency errors carried by the surrounding induction. The operator G is the average slice submeasurement, and $I - G$ is its complement in the Bernoulli polynomial that is compared with the all-outcomes expansion. The symbol $\widehat{H}$ denotes the sandwiched completed-slice product summed over all completed outcomes. Finally, ν_4 is the commutation error of `lem:commute-g-half-sandwich`, and ν_8 is the from-H-to-G error parameter in the lemma under discussion.

The lemma compares the all-outcomes expansion of the pasted submeasurement with a Bernoulli polynomial in G and $I - G$. It states

$$\nu_8 = 46km(\gamma^{1/32} + \zeta^{1/32} + (d/q)^{1/32}). \qquad \texttt{(lem:from-H-to-G)}$$

The proof first proves an adjacent-stage estimate with error $2\sqrt{2\zeta} + 2\sqrt{\nu_4}$. Here ν_4 is the commutation error from the preceding half-sandwich lemma:

$$\nu_4 = 426k^2m(\gamma^{1/16} + \zeta^{1/16} + (d/q)^{1/16}).$$

It then applies the adjacent estimate for k stages.

[1] Tracked as part of the discrepancy audit in GitHub issue #930. In the TeX source consulted here, the statement of the lemma is in `references/ldt-paper/ld-pasting.tex`, lines 1294–1308, and the relevant error calculation is in lines 1370–1376.

2 The arithmetic

The accumulated adjacent-stage error is

$$k(2\sqrt{2\zeta} + 2\sqrt{\nu_4}).$$

Substituting the displayed value of ν_4 gives

$$\begin{aligned} k(2\sqrt{2\zeta} + 2\sqrt{\nu_4}) &= 2k\sqrt{2\zeta} + 2k\sqrt{426k^2m(\gamma^{1/16} + \zeta^{1/16} + (d/q)^{1/16})} \\ &\le 4k\zeta^{1/32} + 42k^2m(\gamma^{1/32} + \zeta^{1/32} + (d/q)^{1/32}) \\ &\le 46k^2m(\gamma^{1/32} + \zeta^{1/32} + (d/q)^{1/32}), \end{aligned}$$

where we use the explicit assumptions

$$0 \le \gamma, \zeta, d/q \le 1 \qquad \text{and} \qquad k \ge 1, \quad m \ge 1.$$

The bounds $\gamma, \zeta, d/q \le 1$ allow the square-root powers to be weakened to 1/32-powers, and $k, m \ge 1$ allow the first term to be absorbed into the final $46k^2m(\cdots)$ bound. The second term is quadratic in k, because the factor k from the telescope multiplies the k already present in $\sqrt{\nu_4}$. The displayed calculation in the paper drops this outer factor from the commutation term.

Thus the statement proved by the displayed telescope has the corrected error

$$\nu_8' = 46k^2m(\gamma^{1/32} + \zeta^{1/32} + (d/q)^{1/32}).$$

This correction does not affect the final pasting theorem: the surrounding completeness argument absorbs the from-H-to-G loss together with the all-outcomes loss into

$$\nu = 100k^2m\big(\epsilon^{1/32} + \delta^{1/32} + \gamma^{1/32} + \zeta^{1/32} + (d/q)^{1/32}\big).$$

The corrected ν_8' has the same quadratic dependence on k as this final parameter.

3 The formal and blueprint statements

The blueprint records the corrected quadratic error in the from-H-to-G lemma and explicitly notes the arithmetic correction.[2] The formal development uses the same corrected quantity.[3] It separates the literal telescoped

[2] See `blueprint/src/chapter/ch09_pasting.tex`, lines 1037–1100.

[3] The corrected error is MIPStarRE.LDT.Pasting.fromHToGError in `MIPStarRE/LDT/Pasting/Statements.lean`, lines 113–124 of the source file. The public lemma MIPStarRE.LDT.Pasting.fromHToG proves the scalar comparison with this error in `MIPStarRE/LDT/Pasting/Bernoulli/FromHToG.lean`. The absorption into ν is carried by the final completeness proof in `MIPStarRE/LDT/Pasting/Bernoulli/Final.lean`.

error from the displayed bound: the telescope contributes k adjacent-stage losses, and the final comparison bounds that telescope by the quadratic ν_8'.

4 Conclusion

The discrepancy is a harmless intermediate arithmetic correction. The linear-in-k value of ν_8 stated in the paper does not follow from the preceding adjacent-stage estimate, because the commutation error ν_4 already contains k^2. The corrected bound is quadratic in k. Since the main pasting theorem already uses a quadratic-in-k error parameter with sufficient slack, the final theorem and the formal statement remain aligned.

References

[JNV+20] Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen. Quantum soundness of the classical low individual degree test, 2020. arXiv:2009.12982.