

The ambient dimension in the commutativity theorem for G

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1 The assertion

We examine the statement of the commutativity theorem for the slice measurements G^x in [JNV⁺20, Section *Commutativity of G*].¹ The source states that (ψ, A, B, L) is an $(\epsilon, \delta, \gamma)$ -good symmetric strategy for the (m, q, d) low individual degree test. It then assumes, on average over $(\mathbf{u}, \mathbf{x}) \in \mathbb{F}_q^{m+1}$,

$$A_a^{u,x} \otimes I \simeq_\zeta I \otimes G_{[g(u)=a]}^x,$$

where G^x is a projective submeasurement with outcomes in $\mathcal{P}(m, q, d)$.

2 The point at issue

The displayed consistency condition is naturally a hypothesis about a strategy for the $(m+1, q, d)$ test. In that test, the point measurement A is indexed by points of $\mathbb{F}_q^{m+1}$, which are written as (u, x) with $u \in \mathbb{F}_q^m$ and $x \in \mathbb{F}_q$. A strategy for the (m, q, d) test has point measurements indexed by $\mathbb{F}_q^m$ instead, so $A_a^{u,x}$ is not the corresponding point measurement.

The surrounding argument also uses the theorem in the successor step: the slice measurements G^x live in dimension m , and they are pasted to produce a measurement in dimension $m+1$. Thus the intended ambient strategy is the $(m+1, q, d)$ strategy, while the slice family remains a family of m -variable polynomial submeasurements.

3 The formal and blueprint statements

The blueprint states the corrected ambient dimension: in `blueprint/src/chapter/ch08_commutativity.tex`, the theorem `thm:com-main` assumes an $(\epsilon, \delta, \gamma)$ -good symmetric strategy for the $(m+1, q, d)$ low individual degree test, and takes projective submeasurements G^x in `PolySub( $m, q, d$ )`.

¹Tracked in [GitHub issue #930](#). The paper passage is `references/ldt-paper/commutativity-G.tex`, lines 228–257.

The formal theorem uses the same convention.² It proves the commutativity conclusion for the full slice products

$$G_g^x G_h^y \otimes I \approx_\nu G_h^y G_g^x \otimes I, \quad \nu = 30m \left(\gamma^{1/4} + \zeta^{1/4} + (d/q)^{1/4} \right),$$

with the same three hypotheses of consistency with A , strong self-consistency, and boundedness.

4 Conclusion

The occurrence of (m, q, d) in the source statement of `thm:com-main` is a harmless dimension typo. The theorem is used with an ambient $(m+1, q, d)$ strategy and a family of m -dimensional slice measurements. The blueprint and the formal development use this corrected statement. No Lean theorem statement or proof needs to change.

References

- [JNV⁺20] Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen. Quantum soundness of the classical low individual degree test, 2020. arXiv:2009.12982.

²The declaration is `MIPStarRE.LDT.Commutativity.comMain` in [MIPStarRE/LDT/Commutativity/Main/Results.lean](#). Its parameter `params` describes the slice dimension m , while the strategy has type `SymStrat params.next`, where `params.next.m = params.m+1`.