Issue 1099: The Sharper Local Repair for the Line-169 Transport Step

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1 At a glance

- **Difficulty.** Medium: the mathematical correction is elementary, but it occurs inside the quantitative projectivization route for MIP-StarRE.LDT.Test.mainFormal.
- **Estimated weight.** The correction is already formalized by one completion monotonicity theorem and two transport theorems; no new theorem interface is expected, and future maintenance should be at the level of tens of lines rather than a new construction.
- **Mathlib/project split.** About **20% Mathlib** and **80% project-local**: Mathlib supplies positivity and real-arithmetic facts, while the tensor-placement, consistency, and completion statements are project-local.
- **Key Mathlib inputs.** Positivity of sums and square roots, elementary real arithmetic, and order reasoning for positive semidefinite matrix operators.

2 Key theorem forms

The formalization isolates the following statements.

1. **Source theorem form.** The paper uses the line-169 transport step as if completion of the orthonormalized projective submeasurement caused no additional loss beyond the original consistency error ζ_1.

2. **Corrected theorem form.** The formalized repair compares with the pre-completion projective submeasurement, paying the square-root loss at the orthonormalization scale $\epsilon = 100\zeta_1^{1/4}$, and then proves that canonical completion does not decrease the diagonal match mass.

3. **Downstream theorem form.** The route consumed by MIPStarRE.LDT.Test.mainFormal gives $Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+10\zeta_1^{1/8}} I \otimes G_g^{\mathrm{B}}$ and the Bob-side mirror.

3 Purpose

This note records the sharper local correction to the line-169 transport step in the main-induction projectivization chain of [JNV+20].[1] The broader discrepancy note `issue-1099-line169-triangle-sub-loss.tex` explains why the printed paper argument cannot yield the exact error ζ_1. The present note isolates the sharper local repair that is now used by the active MIPStarRE.LDT.Test.mainFormal assembly. The older exact line-169 match-mass interface has been retired; the current formalization keeps the repaired transport helper in the final theorem route instead.

The point of this repair is that the generic corrected route

$$\zeta_1 + \sqrt{\zeta_2}$$

is mathematically valid but quantitatively wasteful, because ζ_2 is the *post-completion* error. If one compares with the orthonormalized projective submeasurement *before* completion, then the square root lands on the smaller orthonormalization error instead, producing the sharper bound

$$\zeta_1 + 10\zeta_1^{1/8}.$$

4 The setup

At this stage of the induction, the argument has already produced two polynomial-outcome measurements G^{A} and G^{B} satisfying the line-130 consistency estimate

$$G_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1} I \otimes G_g^{\mathrm{B}}. \qquad \text{(eq:G-self-consistency)}$$

[1] Tracked by GitHub issue #1099. The surrounding projectivization argument is in `references/ldt-paper/inductive_step.tex`, lines 130–185. The specific line-169 replacement step is lines 167–173.

Orthogonalization then supplies projective submeasurements P^{A} and P^{B} with

$$G_g^{\mathrm{A}} \otimes I \approx_{100\zeta_1^{1/4}} P_g^{\mathrm{A}} \otimes I,$$

and similarly on Bob's side.

Write

$$\epsilon := 100\zeta_1^{1/4}.$$

Then the orthonormalization estimate is simply

$$G_g^{\mathrm{A}} \otimes I \approx_{\epsilon} P_g^{\mathrm{A}} \otimes I.$$

Afterwards one completes P^{A} to a projective measurement Q^{A} by adding the missing mass $I - \sum_g P_g^{\mathrm{A}}$ at a distinguished outcome.

5 The sharp local correction

The correction proceeds in two steps.

Step 1: compare before completion

Fix the partner measurement G^{B}. Proposition `prop:easy-approx-from-approx-delta` bounds the change in diagonal match mass when one replaces G^{A} by a $\approx_\epsilon$-close family. Specialized to the constant single-question setting, it gives

$$\left| \sum_g \langle \psi, (G_g^{\mathrm{A}} \otimes G_g^{\mathrm{B}})\psi \rangle - \sum_g \langle \psi, (P_g^{\mathrm{A}} \otimes G_g^{\mathrm{B}})\psi \rangle \right| \leq \sqrt{\epsilon}.$$

Since line 130 says that the first match mass is at least $1 - \zeta_1$, one obtains

$$\sum_g \langle \psi, (P_g^{\mathrm{A}} \otimes G_g^{\mathrm{B}})\psi \rangle \geq 1 - \zeta_1 - \sqrt{\epsilon}.$$

Equivalently,

$$P_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1 + \sqrt{\epsilon}} I \otimes G_g^{\mathrm{B}}.$$

Step 2: complete without further loss

Now pass from P^{A} to the completed projective measurement Q^{A}. The completion changes exactly one outcome by adding the positive operator

$$I - \sum_g P_g^{\mathrm{A}}.$$

Therefore the diagonal match mass against the fixed partner measurement G^{B} can only increase. No new negative term appears in the consistency defect. Hence

$$Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+\sqrt{\epsilon}} I \otimes G_g^{\mathrm{B}}.$$

Substituting the value of ϵ gives the explicit form

$$Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+\sqrt{100\zeta_1^{1/4}}} I \otimes G_g^{\mathrm{B}},$$

that is,

$$Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+10\zeta_1^{1/8}} I \otimes G_g^{\mathrm{B}}. \qquad \text{(sharper repaired line 169)}$$

The Bob-side mirror is identical:

$$I \otimes Q_g^{\mathrm{B}} \simeq_{\zeta_1+10\zeta_1^{1/8}} G_g^{\mathrm{A}} \otimes I.$$

6 Why this is sharper than the generic correction

The generic corrected route applies `prop:triangle-sub` only after the completion estimate

$$G_g^{\mathrm{A}} \otimes I \approx_{\zeta_2} Q_g^{\mathrm{A}} \otimes I,$$

and therefore yields

$$Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+\sqrt{\zeta_2}} I \otimes G_g^{\mathrm{B}}.$$

This is correct, but ζ_2 is already the larger post-completion scalar. In contrast, the local repair takes the square root earlier, at the smaller orthonormalization scale $\epsilon = 100\zeta_1^{1/4}$. Thus

$$\zeta_1 + 10\zeta_1^{1/8}$$

is strictly better than

$$\zeta_1 + \sqrt{\zeta_2}$$

for the present cascade.

This quantitative distinction explains why the sharper repair was preferable to the generic corrected route. It is also the correction used in the active MIPStarRE.LDT.Test.mainFormal proof route: the square-root loss is taken before completion, and completion is then handled by the separate match-mass monotonicity lemma.

7 Relation with the formal development

The formal development still contains the lower-level local repair lemmas.[2]

In the current code, the paper-facing theorem MIPStarRE.LDT.Test.mainFormal has a direct proof gap rather than a separate bridge or residual hypothesis. The final transport used once the projective-completion witness has been constructed is factored through MIPStarRE.LDT.Test.mainFormal_ofProjectiveCompletionTransportWitnes The older repaired base-case assembly theorem has been removed from the current MainFormal route, and the repaired line-169 Step-8 absorption helper has also been deleted as dead code. The generic corrected route $\zeta_1 + \sqrt{\zeta_2}$ remains mathematically available, but it is not the active route for the present formal theorem.

8 Conclusion

The sharper local correction is obtained by inserting the square root at the orthonormalization stage rather than at the completion stage. Writing $\epsilon = 100\zeta_1^{1/4}$, the corrected line-169 transport is

$$Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+\sqrt{\epsilon}} I \otimes G_g^{\mathrm{B}},$$

equivalently,

$$Q_g^{\mathrm{A}} \otimes I \simeq_{\zeta_1+10\zeta_1^{1/8}} I \otimes G_g^{\mathrm{B}}.$$

This does not recover the paper's printed exact ζ_1 line, but it is a strictly sharper correction than the generic route $\zeta_1+\sqrt{\zeta_2}$ and is sufficient to preserve the original final error envelope in the current formal base-case assembly.

References

[JNV+20] Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen. Quantum soundness of the classical low individual degree test, 2020. arXiv:2009.12982.

[2] The general pre-completion transport statements are MIPStarRE.LDT.MakingMeasurementsProjective.ProjectivizationLine169Repair.leftConsistency_of_completion_and_sdd and MIPStarRE.LDT.MakingMeasurementsProjective.ProjectivizationLine169Repair.rightConsistency_of_completion_and_sdd in `MIPStarRE/LDT/MakingMeasurementsProjective/ProjectivizationChain/Line169Repair.lean`. Specializing the error to MIPStarRE.LDT.MakingMeasurementsProjective.orthonormalizationError gives MIPStarRE.LDT.MakingMeasurementsProjective.ProjectivizationLine169Repair.leftConsistency_with_orthonormalization_loss and its Bob-side mirror.